

## **Title**

Developing Algebraic Thinking in Grades 3-5

## **Target Audience**

This course is intended for pre-service and in-service teachers, grades 3-5.

## **Course Description**

This course focuses on strategies for incorporating algebra in the classroom. Learners will explore the algebraic content present in the activities they currently use with students, and develop ways to encourage students to think algebraically. Learners will solve mathematical problems in order to better understand patterns, analyze situations using algebra, and practice representing relationships with mathematical models. As part of the course assignments, learners will practice their newly-developed strategies on 3-5 grade students, monitoring their learning process and assessing their understanding of algebra.

This course is aligned with the recommended standards of the National Council of Teachers of Mathematics (NCTM). By analyzing these standards and principles, learners will have a good understanding of what third, fourth, and fifth graders need to learn to prepare for more advanced algebra in middle school. As a final project, learners create a sequence of three problems whose use in the classroom will help students develop their algebraic thinking.

## **Instructor/Facilitator**

See instructor/facilitator sheet

## **Credits**

To be determined by college or university

## **Objectives**

Learners will:

- Gain a deeper understanding of the potential for algebraic thinking in many activities involving number sense in grades 3-5
- Experience listening with an analytical ear to children's thinking to better assess what they do and do not know
- Learn strategies for bringing out algebraic thinking in the classroom

## **Outline of Content and Assignments**

After previewing the course introductory information, learners will proceed to the Assignments section to complete the following six sessions, working through each part in order. In these sessions, they will solve mathematical problems that involve algebraic thinking. They will use that experience as the basis for understanding what students do and do not know about such problems, and for developing strategies that can lead them to further algebraic thinking. Learners will also read NCTM's Principles and Standards of School Mathematics in order to see how the kinds of thinking students are doing in grades 3-5 will help them think more algebraically in middle school. As a final task, they will generate and justify a sequence of problems that can help students develop ways of thinking algebraically.

Part 1: Orientation (Session 1)

Learners will:

Test their computers

- Run "The Wizard"
- Install all required plug-ins to run PBS TeacherLine courses

Become familiar with the course website

- Click on the different sections of the course
- Click the "View Video" button to watch a short, informative video about preparing to think like a learner, as well as a teacher, for this course
- Download a copy of the Learning Log, used to describe how the math problems presented in this course were solved
- Open the Learning Log to become acquainted with the kinds of questions that you will answer throughout the course.
- Print out the Digital Drop Box instructions for later use in Sessions 3 and 6 (used to post Learning Log to the facilitator).

Read

"Building a Foundation for Learning Algebra in the Elementary Grades," from the National Center for Improving Student Learning and Achievement in Mathematics and Science, describing research on student thinking. It illustrates how students as young as first and second graders can think in an algebraic manner.

Collaborate

- Introduce yourself on the discussion board
- Learn how to communicate by posting messages on the board

Prepare

- A notebook to be used for activities. You may want to have graph paper for your drawings and diagrams. A notebook will keep you organized.
- A group of students in grades 3, 4, or 5, either from your classroom or elsewhere. They will be needed to test math problems suggested in this course (after Sessions 2 and 3).

Preview

- The "homework" assignments. These are indicated 3 times in the sessions under the title "Looking Ahead" and will help the learners prepare for upcoming activities.

Part 2: Getting Started with Patterns and Relations (Session 2)

Learners will:

Read



- National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics (PSSM 2000)*:
  - General standards on algebra—sections on patterns in the number system; finding and generalizing patterns. (pp. 37-40)
  - Grade 3-5 standards—section on patterns, relationships, and functions. (pp. 158-163). The standards emphasize the role of patterns as a basic foundation for mathematics. Without patterns, there would be no mathematics; even the simplest mathematical activity, counting, relies on finding and extending a pattern. This session examines patterns of multiples and challenges learners to think about more than one multiples pattern at once.

Solve

- The “Mystery Multiples Problem,” using the interactive as an aid.

Create

- Three Mystery Multiples Problems to use with the group of students they have chosen to work with, using the interactive to help construct them.

Discuss

- The way one of the three problems were approached on the Discussion Board.
- A personal answer to the following question: “Were there any surprises for you in solving these?” Share something that caught your attention while you were working on these problems.

Preview

- The material preceding Session 4:
  1. Pose the problems created to the group of students you have decided to work with
  2. Provide students with the 100s charts to work on
  3. Observe their work carefully and ask questions to further their thinking
  4. Keep detailed notes in their notebook on students' work on one problem and on how they interacted with the students while they were working on that problem
  5. Save any written work they do to reflect on later

Part 3: Properties of Our Number System (Session 3)

Learners will:

Read

- "Properties of Multiplication" from *Relearning to Teach Arithmetic: Multiplication and Division* by Susan Jo Russell.
- The description of the “properties of the number system.”

Solve

- The “Cluster Problems,” using the interactive and reading some commentary on using arrays to teach multiplication.

Record in the learning log

- The steps taken to solve each of these cluster problems and how the results of some of the earlier problems were used to figure out the last one. How were the results of the earlier calculations combined to construct the final one?

Send

- The learning log to the facilitator using the Digital Drop Box.

Participate in an online discussion

- Regarding this use of cluster problems with the distributive property. Some people find it to be very helpful, while others do not. What do you see as the advantages or disadvantages of each and why? Post comments on the Discussion Board, responding to at least two other posts.

Preview

- The material preceding Session 5:
  1. Make up at least one cluster problem that is appropriate for the students you have chosen to work with.
  2. Between now and Session 5, pose the problems(s) to the students.
  3. Observe their work carefully and ask questions to further their thinking. Keep detailed notes in your notebook on at least one student's work and on how you interacted with that student.
  4. Save any written work that your students did to reflect on later.

Part 4: Reflecting on Students' Ideas, 1 of 2 (Session 4)

**(Note: This session has 2 discussion threads)**

Learners will:

**(Discussion A)** Participate in an online discussion:

- Using the Discussion Board to report on your experience working with your students on the Mystery Multiples Problems. (Refer to Session 2 for the assignment description.) Include the following:
  - A description of one of the students' approaches to a problem, highlighting the mathematics the student used at each step. Include the questions you asked and how the student responded.
  - A description of how this student showed what he or she understood about multiples. How did your questions help this student build on the knowledge he or she already had? How did you decide whether to ask a question or provide an answer in this case?
- By reading and commenting on two other participants' descriptions of their experience with students. Compare the experience they described to your own. Did their students "get stuck" on similar pieces of mathematical knowledge? Are there similarities in the kinds of questions asked?

Check for feedback

- From the facilitator using the Digital Drop Box.

Watch video

- "Array Cards," of a teacher working with some fourth grade students. Focus on the questions the teacher asks and take notes in your notebook about questions and techniques used.

**(Discussion B)** Participate in an online discussion:

- Using the Discussion Board to report on the "Array Cards" video.
  - What do you think John understood about the distributive property? What further questions could the teacher ask him (or the other students at the table) to extend the algebraic aspects of this problem?
- By reading and commenting on at least two other people's postings.

Part 5: Reflecting on Students' Ideas, 2 of 2 (Session 5)

Learners will:

Watch a video

- Of a teacher working with some fourth grade students on cluster problems. Focus on how the teacher works with the student to figure out how well the student understands the distributive property.

Record notes

- About the teacher's questioning technique and on what she finds out about the student's mathematical thinking, answering these questions: What evidence does she look for? How does she ask questions to elicit this information?

Participate in an online discussion

- Describe your experiences with one of your students working on cluster problems and compare them with what you saw in the video. What did the student know about the distributive property compared to the student in the video? What evidence did you look for and how was this similar to or different from the way the teacher in the video questioned her student?
- By reading and commenting on at least two other people's postings. What are the similarities and differences in the learners' students' mathematical knowledge? What are the similarities and differences in the kinds of questions asked by the learners and what kind of evidence do they look for?

Preview

- The material preceding Session 6 by thinking about "The Piggy Bank Problem" and recording ideas for solving the problem (writing and drawing) in your notebook.

Part 6: Working With Variables and Planning an Activity Sequence (Session 6)

Learners will:

Read

- "Algebra in the Early Grades" by David Carraher, Analucia Schliemann, Barbara M.Brizuela in *Hands On!* Spring 2001.
  - Study the examples of third graders working on the Piggy Bank Problem, noting the algebraic reasoning that children use in solving this problem.
- National Council of Teachers of Mathematics' (NCTM) *Principles and Standards for School Mathematics* (PSSM 2000), Algebra Standard Grades 6-8. (pp. 222-231)

Participate in an online discussion

- Using the Discussion Board, describe your approach to the "Piggy Bank Problem" and how it differed from that of the students in the reading (including any drawings as attachments with descriptions). What algebraic knowledge did you use in solving the Piggy Bank Problem?
- By responding to at least two other posts, discussing the similarities and differences between your approaches to the problem and the algebraic reasoning you used.

**Complete a Final Project**

Bringing together all of the things learned from this course, you will design a series of problems for a Teacher's Guide on algebraic thinking in grades 3-5. You may include any of the kinds of problems you worked with in this course and must:

1. Create a sequence of three problems whose use in the classroom can help students better understand algebraic thinking.
2. Include a detailed rationale for each problem and for the entire sequence.
3. Include in this rationale the mathematical ideas used to try to help students understand. Refer to the PSSM 2000 standards for grades 3-5 or 6-8 to help support the choices.
4. Include some ideas on how students might do these problems and how the sequence takes that into account.

Write

- In the learning log, respond to the following question: What one or two ideas do you take away from this course that you believe will be of the most use for you in your classroom, and why?

Then submit your learning log (including all entries from the entire course as well as facilitator comments), along with your final project, to the facilitator using the Digital Drop Box.

### **Schedule**

This course is scheduled to take approximately 15-20 hours to complete readings, activities, video, assignments, reflections and a final project.

### **Requirements**

Learners are expected to:

- Complete all assignments
- Maintain an online journal
- Participate regularly in discussion boards

### **Evaluation**

Pass/fail upon satisfactory completion of assignments and discussion board participation.

### **Materials (hardware, software, plug-ins)**

#### Technical Requirements

- Word processor
- Internet service provider
- Email

### **Academic Dishonesty Policy**

To be inserted by university institution only