Title

Seeing Math™**:** Proportional Reasoning

Target Audience

This course is intended for pre-service and in-service teachers of mathematics grades 6-12.

Prerequisites

To successfully participate and complete the assignments in this course, the learner must:

- Have knowledge of basic algebra and comfort with fractions, linear functions, slope, and graphing straight lines.
- Be familiar with problems involving ratio, proportion, and scale.

Course Description

The Proportional Reasoning course examines proportional reasoning and its importance in the learning of algebra. The course helps learners understand how students learn proportional reasoning, what difficulties get in their way, and what tools can help with the transition from elementary mathematics to the more complex, proportional thinking of algebra.

The course provides an in-depth examination of the development of proportional thinking, which underlies much of algebraic problem solving. It starts with an orientation week in which learner get to know the online interface and their fellow students, and concludes with a final project to help learner carry what learner learned to their own classroom. Themes in the course include:

- Learning to see mathematical expressions and problems in more than one way
- Distinguishing additive and multiplicative thinking
- Discovering the "hidden picture" of proportional reasoning in algebra, and the importance of learning to frame problems through proportional thinking, rather than just computing an answer
- Finding ways to distinguish proportional reasoning from other approaches
- Representing real-life situations in proportional problems
- Exploring solutions to proportion problems through symbols and graphs

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Explore the relationship between proportional reasoning and algebraic thinking. Learners will be able to:

- Understand why multiplicative thinking is not only basic to proportional reasoning, but essential to algebra
- Analyze the use of proportional reasoning in typical Algebra 1 problems

Understand that students who appear to reason proportionally may in fact be following a procedure without understanding it. Learners will be able to:

- Identify facets of student thinking that may become stumbling blocks to proportional reasoning
- Describe student thinking in terms of four common strategies used in reasoning about proportion problems
- Learn techniques that help to identify true proportional reasoning

Understand methods for developing students' proportional reasoning. Learners will be able to:

- Identify situations that require reasoning beyond additive thinking, and create bridges to students' development of multiplicative thinking
- Identify, in curriculum materials, approaches that confirm, extend, or hinder the development of proportional reasoning skills

Outline of Content and Assignments

After previewing the documents in the Course Information area, learners will proceed to Course Content to complete the five sessions, working through each session in order. Throughout the sessions, learners are asked to articulate their ideas in various forms: they are encouraged to reflect on their ideas and experiences in their online journal; the discussions in the discussion forum are designed to allow learners to glean information from other learners' experiences. As a final project, learners create a slide show which answers an essential question from their curriculum, incorporate it into a lesson plan, and implement it in their classroom. They also design a multimedia project for their students in which they will answer an essential question in the curriculum. Finally, they will reflect on the lesson implementation.

This five-week course is taken entirely over the Internet. Expect to spend 4-6 hours per week completing assignments and discussions, and to log in to class and submit work or join discussions at least three times a week. Each week learners will complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Session 1: Orientation
In this session learners will begin to think about ways to identify teaching that emphasizes higher-level thinking and leads to understanding or insight among students. They will also begin to think about ways to use multimedia technology to facilitate this higher-level understanding and insight.

Learners will:

Read
- *Landscape of Learning* which addresses the underlying principles behind the learning in this course and what they should expect
- *Shifting Perspectives*, a discussion about the importance of being able to view mathematical expressions in more than one way.

Complete activities and assignments
- Create personal homepage providing a biographical sketch

Write in online journal
- Reflect on insights and ideas related to the two discussion readings: Landscape of Learning and "Shifting Perspectives".

Participate in online discussion

- Post messages in the Discussion Board, Session 1, *Getting to Know You.*
- Share thoughts on "The Landscape of Learning" and "Shifting Perspectives" in the Discussion Board, Session 1, *Shifting Perspectives and Landscapes.*

Session 2: Math Focus

Learners will look at how typical Algebra 1 curricula address proportional reasoning. Then they will then address the subject matter by working through a set of ratio and proportion problems and reflecting on the reasoning used to solve them. Next, they will look at how some rules students acquire in elementary mathematics can actually get in the way of learning Algebra 1. Finally, learners will explore the challenges they face in helping their students think proportionally—as they consider why ratio and proportion are such difficult concepts to grasp.

.

Learners will:

Read

- *Snapshots from the Curriculum*: An overview of how curricula address proportional reasoning.
- *Observing Your Processes*: Guidelines on observing personal problem solving.
- *Connecting Personal Experiences and Research* which presents obstacles that may unwittingly be created in our elementary math education
- *All out of Proportion* which addresses why proportional reasoning concepts are difficult for students to grasp.

Write in journal (not required)

- Reflect on insights gained from solving multiple and diverse problems requiring proportional reasoning.

Participate in an online discussion:

- Share solutions and comments in the Discussion Board, Session 2, *Diving In: Proportional Reasoning*
- Share comments in the Discussion Board, Session 2, *All Out of Proportion.*

Complete activities and assignments

- Diving In: Proportional Reasoning - Solve problems that compare phone card services, peanut prices, and dimensions for theater set design.

Session 3: Student Thinking

This session focuses on the mystery of student thinking. Learners will observe students at their mathematical work, and take a closer look at the many paths to proportional reasoning.

Videos show students working on the *Diving In* activities that learners tackled during Session 2. Learners are to listen and observe closely how students approach the same problems and look for ways to support their students' understanding. Comments from mathematics specialist, Dr. Richard Lesh are also presented to assist learners in looking at ways of assessing students' development of proportional reasoning.

Learners will:

Read

- *Meet the Students –Part 1* which provides background information about the students in the first set of videos.
- *Assessing Proportional Reasoning* – Looking at examples of true proportional reasoning and techniques for assessing student thinking.
- *Recognizing Contrasting Strategies* -Examine other strategies that students may apply to proportional reasoning problems.
- *Meet the Students –Part 2* which provides background information about the students in the second set of videos Read about the students they will meet in the second set of videos.

View videos:

- Observe students working with on the Phone Card and Peanuts problems.
- Watch as mathematics specialist, Richard Lesh, discusses the underlying mathematical themes in the students' work.
- Take notes on their observations as they preview the videos.
- Observe students working on the Showtime problem.

Write in Journal (not required):

Reflect on what learners considered interesting or significant in the videos

- Which strategies (including the truly proportional) resonate with their approaches to the problems? Did learner use more than one strategy? Do learners find themselves favoring one approach over another?
- Have learner noticed their students using any of these strategies? What successes or difficulties have learner observed?
- Some teachers like to use a structured method, or an organizer—such as lists or boxes—or even a proportion format with "plug-in" values. How can learner use some of the ideas in this course to discover whether their students really understand the *proportions* in a problem, and are not just blindly carrying out the mechanics?

Participate in an online discussion:

- Share insights with their colleagues in the Discussion Board, Session 3, *Observing Student Thinking: Part 1.*
- Share insights with their colleagues in the Discussion Board, Session 3, *Recognizing Contrasting Strategies.*
- Share insights with their colleagues in the Discussion Board, Session 3, *Observing Student Thinking: Part 2.*

Session 4: Your Classroom

Math specialist Richard Lesh discusses some issues from the classroom—in particular, what approaches work best for teaching math, and in what balance? How important is it to allow students time to grapple with the math: to express, test, and revise their own ideas? How important is it to guide directly and ensure that students arrive, in the end, at approved mathematical ways of solving problems?

Learners will look through their curriculum for two problems to adapt for their students, using what they have learned in this course. They will post their results and observations, and study the problems their colleagues have designed. They will then look at graphing and how it can connect to proportion. Finally, learners can incorporate some of the activities in their instructional programs.

Learners will:

Watch Video:

- Watch as mathematics specialist Richard Lesh discusses Richard Lesh comments on the underlying mathematical themes and approaches to teaching.

Write in Journal:

- How might graphing help to teach proportional reasoning problems in a way that supports deeper, nonprocedural understanding?
- At any point did learner feel tempted to draw a line (connecting the points) and calculate the slope? (Learner could try that now.) What were the slopes and what did they reflect? How might the definition of slope as a "rate of change" build a deeper connection to proportion?
- How might this graphing technique have value as a self-check for students?
- Would the graphs of all proportional relationships form a line? Why? Explain reasoning.

Participate in an online discussion:

- Share insights with colleagues in the Discussion Board, Session 4, *Links to Algebra* on two of the issues from their reflections.

Complete activities and assignments:

- Explore the connections between proportional relationships and graphing by extending to the Peanuts problem to include a graphing approach.
- Review curriculum and adapt two problems students that build on the themes of this course.
- Incorporate in the instructional program some of the problems worked in this course.

Session 5: Your Plan

In this session learners will look back over the landscape of ideas they explored during this course. They will review their thoughts and records to consolidate their learning experience to create a project that will integrate the algebraic concepts developed throughout the course into their teaching practice. Learners will also celebrate their achievements and say goodbye to their peers and facilitator.

Learners will:

Read:

- A review of major topics addressed in this course

Complete activities and assignments:

- Create either a lesson plan or action plan for applying what was learned to their instructional program:
  - Lesson plan – Select a specific activity (such as one of the "For Your Students" activities) that facilitates having students share mathematical ideas. Modify it to address the learning styles and characteristics of their students.
  - Action plan – Select a specific action or instructional strategy learner want to address, such as focusing on specific kinds of questions that elicit student thinking or specific personal activities to cultivate their listening skills

Participate in an online discussion:

- Post in the Discussion Board, Session 5, *Gallery of Plans-learners' completed plans*

Write in online journal (not required):

- Review written work and memories of what was learned and record a personal self-assessment and reflection.

Schedule

This course is designed to be a 30 hour course conducted over 5 weeks. Learners will spend 4 to 6 hours per week to complete assignments such as solving problems, observing videos, reading,

adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Requirements

Learners are expected to:

- Complete all assignments.
- Maintain an online journal.
- Participate and actively engage in discussions with fellow learners while contributing to the social construction of knowledge.
- Be self-directed and self-motivated.
- Ask for assistance when they need it.

Materials (hardware, software, plug-ins for Windows and Macintosh)

Operating System
For the best experience, use the newer operating systesm: Mac OS X, Windows 98, Windows 2000 and Windows XP. Additional operating systems (for example Linux) appear to work, but are not tested. Mac OS 9 does not support a current version of Java, which is needed to use the interactives.

Browser
Use Internet Explorer, Mozilla, or Netscape with Windows operating systems. MAC users should use Netscape or Mozilla. Browser must have cookies enabled to support course login.

Video Players

One of the following video players is required in order to view the videos. Seeing Math recommends QuickTime.

- QuickTime
- RealPlayer
- Windows Media Player

Java
This course contains "interactives" — software applications that focus on one particular math concept. These require Java 1.3.1 or higher.

Word processor
Internet service provider
E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course can be taken for graduate credit on a pass/fail basis, or for a letter grade and graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

Last Update: December 2, 2005

Rubrics for Discussion

The assessment rubrics fall into two categories: discussions and activities. Learners read these rubrics to get the "big picture" perspective of what's expected. They then refer to them from time to time during the course to remind them of the target, and to use as a self-assessment tool.

In an online course, participation means posting. Most activities in this course require you to share your thoughts on a subject (such as a reading or a video), or to complete a hands-on assignment and discuss the experience with your peers. This collaboration leads to insights unavailable to individuals alone—we all learn together.

The facilitator will look for ***frequent*** and ***appropriate*** contributions to class discussions from all participants. "Frequent" means posting on at least three days each week. "Appropriate" is based on the level of contribution as a whole (rather than allotting specific points for content, style, particular solutions, etc.). The following characteristics make up an excellent body of discussion contributions:

- Is grounded in the ideas, readings, and activities of the course.
- Connects to and builds on the ideas of others, and advances the collective thinking about content and pedagogy.
- Shows respect for and integrates multiple views (even views that at first appear contradictory or unrelated).
- Achieves or reaches toward new insights about mathematics and teaching.
- Takes risks by sharing tentative or newly formed ideas, mistakes, or misconceptions.
- Expresses content clearly.
- Makes skillful connections between natural language, mathematical language, and student thinking.
- Elicits reflection and responses from other participants.
- Questions other participants in order to clarify and extend own ideas.

Rubric for Mathematical and Pedagogical Activities

Assignments ask learners to post written work in the course—for instance, when they solve a problem and describe their thought processes in working towards a solution. They are asked to wrestle with a math problem, interactive, or ideas. Then share this work with their facilitator and peers as a post in the Discussion Board.
The facilitator measures learners' effort, care, and understanding in reading and carrying out the assignments using the following criteria: The learner:

- Posts clear and detailed reports on assignments and observations of own learning processes.
- Focuses not on the "right answer," but on experiencing and observing learning processes.
- Makes connections among more than two representations (real-life, symbolic, graphic, numeric).
- Considers what different representations contribute to one's own and students' learning of algebra.
- Generates different real-life situations for the same mathematical setting, and conversely, generates different mathematical models to describe variations on the same real-life situation.
- Makes connections among mathematical concepts and describes them clearly.
- Explores the consequences of those connections to understanding and teaching mathematics.
- Clearly identifies, describes, and justifies the strategies used to solve problems.

While these rubrics may seem ambitious, learners are not required to meet every criteria for each assignment. The facilitator will apply individual criteria as necessary (for instance, not all activities require learners to use multiple representations of math concepts). Learners use these as a general guide to gauge the quality of their work.

Learners are also encouraged to keep a journal of their thoughts and rough drafts which serves as an automatic record of their work.