Title

Seeing Math: Linear Equations

Target Audience

This course is intended for pre-service and in-service teachers of mathematics grades 6-12.

Prerequisites

To successfully participate and complete the assignments in this course, the learner must:

- Have an understanding that some functions have more than one solution set.
- Have practice transforming word problems into functions.

Course Description

Most Algebra 1 curricula introduce linear equations before linear functions. In this course, functions are discussed in order to lay the groundwork for learners to:

- View an equation as a function to which a particular value has been assigned
- Gain a deeper understanding of equivalence
- Make the connection between symbolic, graphic, and other representations of equations
- Evaluate how this approach of teaching equations under the umbrella of functions may help students

In this course learners use a variety of approaches, including an interactive tool, to investigate the meaning of processes used to solve equations. They also come away with a tangible benefit—interactive software and activities to use with their students. Learners can use these within the course, so they will be thoroughly familiar with them. (If they lack computer resources in their classroom, alternative activities that do not require computers are provided.)

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Understand the rationale behind the rules of symbol manipulation that maintain an equality or corresponding inequality. The objectives for the participant are to be able to:

- Understand the relationship between variables by dynamically changing the values of *x* and *y*
- Develop solution techniques (including techniques for solving simultaneous equations) by comparing standard symbolic operations to graphic and area representations of equations

Deepen the distinction between equivalence of function and equality of value. The objectives for the participant are to be able to:

- Interpret the "=" sign in terms of equivalence, description of state, inviting a calculation, or defining a quantity

- Distinguish *x* as a variable (in a function, where different values of *x* result in different values of *y*) from *x* as an unknown (in an equation, to be solved for *x*)

Gain facility in moving easily between symbolic and graphic techniques for solving equations and inequalities, whether presented in symbolic or story (text) form. The objectives for the participant are to be able to:

- Clarify what values change (or do not change) during standard operations for solving equations, by using graphic techniques to represent the operations
- Solve word problems with greater skill through the use of graphs to represent the problems

Outline of Content and Assignments

After previewing the documents in the Course Information area, learners will proceed to Course Content to complete the five sessions, working through each session in order. Throughout the sessions, learners are asked to articulate their ideas in various forms: they are encouraged to reflect on their ideas and experiences in their online journal, and they take part in online discussions designed to allow them to glean information from other learners' experiences.

This five-week course is taken entirely over the Internet. Learners should expect to spend 4-6 hours per week completing assignments and discussions, and to log in to class and submit work or join discussions at least three times a week. Each week learners will complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Learners will also come away with a tangible benefit—interactive software and activities to use with students. These tools will be used within the course, such that learners are thoroughly familiar with them. In addition, learners are provided with alternative activities that do not require computers in case computer resources are not available for classroom use.

Session 1: Orientation

Much of the *Orientation* session is spent getting to know the course and meeting colleagues online. Learners also read about the approach to learning and teaching algebra implemented in this course.

Learners will:

Read

- Information about the syllabus, facilitator, rubrics and how work will be evaluated, and tips for successful online discussions.
- A summary of the goals and objectives for the course.
- *The Landscape of Learning* - A discussion on the underlying principles behind the learning in this course and what they should expect.
- *The Landscape of Algebra* – A discussion about the structure of functions that underlies algebra, and how this structure can help students make connections
- *Nouns, Verbs and Mathematics* - A discussion about the importance of being able to view mathematical expressions in more than one way.
- *Let's Discuss Approaches to Algebra* – Discussion questions related to the three primary reading assignments.
- *Journal* – About the value of keeping a record of one's own thoughts and problem solving during the course.

Complete activities and assignments

- Create personal homepage providing a biographical sketch.

Write in journal *(Note: Keeping a journal record is an ongoing assignment referred to within course activities, but the journal is not handed in or evaluated by the facilitator)*:

- Reflect on insights and ideas related to the three readings: *The Landscape of Learning, The Landscape of Algebra* and *Nouns, Verbs and Mathematics*.

Participate in online discussion

- Post messages in the Discussion Board, Session 1, *Getting to Know You.*
- After reading the questions in *Let's Discuss Approaches to Algebra*, share thoughts on *The Landscape of Learning*, *The Landscape of Algebra,* and *Nouns, Verbs* and *Mathematics* in the Discussion Board, Week 1, *Nouns, Verbs, and Landscapes.*

Session 2: Math Focus

This week learners will look at various aspects of equations and functions, including differences between the two. They will focus on ways to give students a better understanding of the process of solving single and simultaneous equations using a variety of methods and representations.

Among the elements learners will investigate are:

- Solving for unknowns using interactive tools and graphical representations
- Finding the point where two linear functions share the same solution set
- Evaluating why the traditional operations used to solve equations maintain the necessary equivalence

Learners will:

Read:

- *Snapshots from the Curriculum* – An article that sets the stage for a different approach to teaching linear equations
- *Observing Your Processes* – Guidelines on observing personal problem solving.
- *Problem Solving Approaches* – Exploring new ways to look at an old problem
- *Solving Systems of Linear Equations* – Discussion about the intersection of functions seen in a new light

Complete activities and assignments:

- *Function Analyzer Warm-up* – Activities to acquaint learners with the Function Analyzer interactive
- *Diving In: Solving Equations from Different Perspectives*

Write in journal-

- Reflect on approaches used to solve the *Diving In* challenges using the Function Analyzer:
- Reflect on the ways that intersections of functions can come into play in the solving of systems of linear equations

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Participate in an online discussion:

- Share solutions, comments and challenges in the Discussion Board, Session 2, *Diving In: Solving Equations from Different Perspectives.*
- Discussion Board, Session 2, *Solving Systems of Linear Equations.*

Session 3: Student Thinking

This week learners visit the theme of real world modeling. Using dynamic models, or models that describe motion, learners will:

- Explore the hurdles that many students face in moving from word problems to symbolic expressions
- Consider strategies for helping students bridge the gap, by focusing on the connections between graphic representations and linear equations

They also review some common difficulties that arise in student understanding of algebra, as revealed by current research. Many are already familiar to you, such as difficulties in distinguishing variables from unknowns, and functions from equations. Learners also look at realistic scenarios in which these confusions arise in the classroom, and gather strategies to help students overcome these hurdles.

In addition, learners will conduct classroom activities with their students, which may help to reveal any points of confusion students may be wrestling with.

Learners will:

Read:

- *Student Thinking: Underlying Research* - A summary of what researchers report on the role of letters in algebra and on the meaning of the equals sign

Complete activities and assignments:

- Read two related classroom scenarios. Record thoughts about what's discussed in the scenarios, as well as responses to specific questions at the conclusion of each scenario. Be prepared to add a summary of their reactions, and responses to the questions raised below (see Write in Journal), to the Discussion Board.

Write in journal

- Reflections for Scenario 1
  - Consider the following questions and write down thoughts:
    - Describe—in one or two sentences for each student—Tyra's, Jenny's, and Louis's thinking in this scenario.
    - How does this scenario reflect what learners may have observed about their own students' interpretations of algebraic letters, including variables vs. unknowns?
    - How do their students' interpretations influence their approaches to problems?

- Reflections for Scenario 2
  - Consider the following questions and write down thoughts:
    - Describe—in one or two sentences for each student—Tyra's, Jenny's, and Louis's thinking in this scenario
    - How does this scenario reflect what learners may have observed about their students' interpretations of the equals (=) sign? What do their students' interpretations lead them to think or do? Share some examples. Be as specific as possible.
    - How might their students' understanding change if they could approach linear equations through the lens of functions?

Participate in an online discussion:

- Post summaries of reflections from reading classroom scenarios in the Discussion Board, Session 3, *Student Thinking: Inside the Classroom. Learners add* a summary of their notes, as well as specific comments about *one or two* ideas that really stood out.

Session 4: Your Classroom

This week the focus is on applying the themes from this course to their classroom. Learners will:

- Examine how their curriculum or textbook uses these themes to support student understanding
- Adapt equation-solving problems from your curriculum to better incorporate the themes of this course
- Examine versions of the *Diving I*n activity from Week Two that they can use in their classrooms

Learners will:

Read:

- *Links to the Curriculum* - key themes intended to support what is likely the need in Algebra 1

Complete activities and assignments:

- Links to the Curriculum - Select two of the underlying themes for closer examination: *Equations and Functions; Equations and Functions; Multiple Representations; or Real World Models and* describe observations with as much background as they can, providing specific examples of the content the curriculum does or does not address.

Write in journal-

- Write up your findings and thoughts from the Links to Curriculum assignment on the two themes selected.

Participate in an online discussion

- Post *two separate* messages, one for each theme, to the Discussion Board, Session 4, *Links to the Curriculum*

Session 5: Your Plan

In this session learners will look back over the landscape of ideas they explored during this course. They will review their thoughts and records to consolidate their learning experience, and create a project that integrates the algebraic concepts developed throughout the course into their teaching practice. Learners will also celebrate their achievements and say goodbye to their peers and facilitator.

Learners will:

Read:

- *Course Summary* - A review of major topics addressed in this course

Complete activities and assignments:

- *Your Classroom Plan* - Create either a lesson plan or action plan for applying what was learned to their instructional program:
  - Lesson plan – Select a specific activity (such as one of the "For Your Students" activities) that facilitates having students share mathematical ideas. Modify it to address the learning styles and characteristics of their students.

- Action plan – Select a specific action or instructional strategy the learner wants to address, such as focusing on specific kinds of questions that elicit student thinking or specific personal activities to cultivate their listening skills

- *Self-Reflection* – Review their course notes on readings and assignments.
- *Aha! Oops! Whew!* – Continue reflecting on the course and choose significant moments of insight into math concepts or teaching ("aha," "oops," or "whew" experiences of learning) and consider how to take these into their teaching.

Write in journal–

- Review written work and memories of what was learned and record a personal self-assessment and reflection.

Participate in online discussions:

- Post the Plan in the Discussion Board, Session 5, *Gallery of Plans*
- Share memorable course experiences and insights with colleagues in the Discussion Board, Session 5, *Aha, Oops, Whew!*

Schedule

This course is designed to be a 30 hour course conducted over 5 weeks. Learners will spend 4 to 6 hours per week to complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Requirements

Learners are expected to:

- Complete all assignments.
- Maintain an online journal.
- Participate and actively engage in discussions with fellow learners while contributing to the social construction of knowledge.
- Be self-directed and self-motivated.
- Ask for assistance when they need it.

Materials (hardware, software, plug-ins for Windows and Macintosh)

Operating System

For the best experience, use the newer operating systems: Mac OS X, Windows 98, Windows 2000 and Windows XP. Additional operating systems (for example Linux) appear to work, but are not tested. Mac OS 9 does not support a current version of Java, which is needed to use the interactives.

Browser

Use Internet Explorer, Mozilla, or Netscape with Windows operating systems. MAC users should use Netscape or Mozilla. Browser must have cookies enabled to support course login.

Video Players

One of the following video players is required in order to view the videos. Seeing Math recommends QuickTime.

- QuickTime
- RealPlayer
- Windows Media Player

Java
This course contains "interactives" — software applications that focus on one particular math concept. These require Java 1.3.1 or higher.

Word processor
Internet service provider
E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course can be taken for graduate credit on a pass/fail basis, or for a letter grade and graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

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Rubrics for Discussion

The assessment rubrics fall into two categories: discussions and activities. Learners read these rubrics to get the "big picture" perspective of what's expected. They then refer to them from time to time during the course to remind them of the target, and to use as a self-assessment tool.

In an online course, participation means posting. Most activities in this course require learners to share their thoughts on a subject (such as a reading or a video), or to complete a hands-on assignment and discuss the experience with peers. This collaboration leads to insights unavailable to individuals alone—we all learn together.

The facilitator will look for ***frequent*** and ***appropriate*** contributions to class discussions from all participants. "Frequent" means posting on at least three days each week. "Appropriate" is based on the level of contribution as a whole (rather than allotting specific points for content, style, particular solutions, etc.). The following characteristics make up an excellent body of discussion contributions:

- Is grounded in the ideas, readings, and activities of the course.
- Connects to and builds on the ideas of others, and advances the collective thinking about content and pedagogy.
- Shows respect for and integrates multiple views (even views that at first appear contradictory or unrelated).
- Achieves or reaches toward new insights about mathematics and teaching.
- Takes risks by sharing tentative or newly formed ideas, mistakes, or misconceptions.
- Expresses content clearly.
- Makes skillful connections between natural language, mathematical language, and student thinking.
- Elicits reflection and responses from other participants.
- Questions other participants in order to clarify and extend own ideas.

Rubric for Mathematical and Pedagogical Activities

Assignments ask learners to post written work in the course—for instance, when they solve a problem and describe their thought processes in working towards a solution. They are asked to wrestle with a math problem, interactive, or ideas. Then share this work with their facilitator and peers as a post in the Discussion Board.
The facilitator measures learners' effort, care, and understanding in reading and carrying out the assignments using the following criteria: The learner:

- Posts clear and detailed reports on assignments and observations of own learning processes.
- Focuses not on the "right answer," but on experiencing and observing learning processes.
- Makes connections among more than two representations (real-life, symbolic, graphic, numeric).
- Considers what different representations contribute to one's own and students' learning of algebra.
- Generates different real-life situations for the same mathematical setting, and conversely, generates different mathematical models to describe variations on the same real-life situation.
- Makes connections among mathematical concepts and describes them clearly.
- Explores the consequences of those connections to understanding and teaching mathematics.
- Clearly identifies, describes, and justifies the strategies used to solve problems.

While these rubrics may seem ambitious, learners are not required to meet every criteria for each assignment. The facilitator will apply individual criteria as necessary (for instance, not all activities

require learners to use multiple representations of math concepts). Learners use these as a general guide to gauge the quality of their work.

Learners are also encouraged to keep a journal of their thoughts and rough drafts which serves as an automatic record of their work.