Title

Seeing Math: Transformations of Linear Functions

Target Audience

This course is intended for pre-service and in-service teachers of mathematics grades 6-12.

Prerequisites

To successfully participate and complete the assignments in this course, the learner must:

- Have familiarity with the slope-intercept form of a line as well as the point-slope form;
- Be able to recognize that changes in the symbolic form of a line affect the graphic representation and vice versa;
- Be able to graph multiple functions on the same axes.

Course Description

In the course, Transformations of Linear Functions, learners build up an understanding of linear functions by observing the relationship between the graphic and symbolic forms of a function. In particular, they will focus on how changing the graphic representation of a function alters its symbolic representation, and vice versa.

Research suggests that when students get comfortable with transformations of linear functions, they improve their mathematical understanding. Some benefits are:

- Making connections between symbolic and graphic representations of functions
- Gaining a deeper understanding of equivalence and slope
- Viewing functions as objects upon which students can perform operations

In this course learners will use a variety of approaches, including interactive tools, to investigate the meaning of processes used to solve equations. Learners will also come away with a tangible benefit—interactive software and activities to use with your students. You use these within the course, so you'll be thoroughly familiar with them. (If learners lack computer resources in their classroom, we provide alternative activities that do not require computers.)

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Observe graphic and symbolic transformations of linear functions. You will be able to:

- Identify the relationships between the graphic and symbolic forms of linear functions
- Interpret changes to the graphic or symbolic forms of linear functions as transformations to the initial function
- Identify and use different symbolic forms for linear functions and equations by exploring the relevance and value of different forms in a variety of situations

Categorize, use, and represent families of linear functions in multiple formats. You will be able to:

- Compare and contrast the symbolic forms of linear functions by identifying elements that make their graphs appear different
- Use interactive software to link specific changes in the symbolic forms of linear functions to corresponding changes in their graphical forms, and vice versa

Interpret the concept of slope in different contexts. You will be able to:

- Interpret slope as a rate of change, and demonstrate rate of change graphically, symbolically, and with virtual manipulatives
- Interpret slope as a geometric ratio of rise over run

Outline of Content and Assignments

After previewing the documents in the Course Information area, learners will proceed to Course Content to complete the five sessions, working through each session in order. Throughout the sessions, learners are asked to articulate their ideas in various forms: they are encouraged to reflect on their ideas and experiences in their online journal; the discussions in the discussion forum are designed to allow learners to glean information from other learners' experiences. As a final project, learners create a slide show which answers an essential question from their curriculum, incorporate it into a lesson plan, and implement it in their classroom. They also design a multimedia project for their students in which they will answer an essential question in the curriculum. Finally, they will reflect on the lesson implementation.

This five-week course is taken entirely over the Internet. Expect to spend 4-6 hours per week completing assignments and discussions, and to log in to class and submit work or join discussions at least three times a week. Each week learners will complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Session 1: Orientation
In this session learners will begin to think about ways to identify teaching that emphasizes higher-level thinking and leads to understanding or insight among students. They will also begin to think about ways to use multimedia technology to facilitate this higher-level understanding and insight.

Learners will:

Read
- *Landscape of Learning* which addresses the underlying principles behind the learning in this course and what they should expect
- *The Landscape of Algebra* which addresses the structure of functions that underlies algebra, and how it can help students.
- *Nouns, Verbs, and Mathematics* which provides an imaginative way to understand functions

Complete activities and assignments
- Create personal homepage providing a biographical sketch

Write in online journal
- Reflect on insights and ideas related to the two discussion readings: Landscape of Learning and "Landscape of Algebra and Nouns, Verbs and Mathematics".

Participate in online discussion

- Post messages in the Discussion Board, Session 1, *Getting to Know You.*
- Share thoughts on the two "Landscapes" and the "Nouns and Verbs" reading in the Discussion Board, Session 1, *Nouns, Verbs, and Landscapes.*

Session 2: Math Focus

The focus of this week is understanding that transforming a function is more than moving individual points on a line. It is recognizing that changing the symbolic representation (equation) of a function changes its graphic representation. Learners will investigate transformations of linear functions using an interactive tool called the Linear Transformer.

Learners will:

Read:

- *Snapshots from the Curriculum*: An overview of the meaning of "transformation."
- *Observing Your Processes*: Guidelines of what to observe as learners do the activities.
- *Connecting Personal Experiences and Research* which presents obstacles that may unwittingly be created in our elementary math education
- *All out of Proportion* which addresses why proportional reasoning concepts are difficult for students to grasp.

Write in journal:

- Review notes from experiences engaging in the Starburst Problem challenges and address the following:
  - How did this exercise influence or support your understanding of slope and intercepts?
  - How were the elements of the symbolic expression highlighted or made tangible by transforming the lines?
  - As you worked through the challenges, when did you choose to manipulate the symbolic or graphic representations?
  - Many algebra curricula focus more on symbolic expressions than on graphic representations. When you were working through these activities, did you have a preference for manipulating one representation over another (symbolic or graphic)? How do your own choices compare to the preferences you tend to emphasize in your classroom?
- Review notes from experiences engaging in the Diamond within A Diamond Problem challenges and address the following:
  - How did your experience with this activity compare to your experience with the Starburst problem?
  - What patterns did you notice as you transformed the diamond-within-a-diamond pattern? How did noticing connections between the symbolic and graphic representations of the functions that make up the diamond help you solve each challenge?
  - When writing the symbolic representation of a function, the notation $f(x)$ can be used interchangeably with y. Students often have trouble interpreting the label $f(x)$ because they mistake its meaning for "f multiplied by x." However, function notation can be a very useful way to express transformations of functions. For example, translating a function $f(x)$ to the left 1 unit can be expressed as $f(x + 1)$. Think about how you might symbolically represent the transformations you made to the diamond-within-a-diamond pattern. How might you use this notation with your students? What ambiguities might arise? What concepts or areas of difficulty might this notation clarify for them?

Participate in an online discussion:

- Share comments in the Discussion Board, Session 2, *Diving In: The Starburst Problem.*
- Share comments in the Discussion Board, Session 2, *Diving In: A Diamond within a Diamond.*

Complete activities and assignments

- *Warm-up: The Linear Transformer:* Activities for getting to know the Linear Transformer software tool
- *Diving In: The Starburst Problem* - Work on activities using the Linear Transformer
- *Diving In: A Diamond within a Diamond* - Work on activities using the Linear Transformer

Session 3: Student Thinking

During this session, learners will consider the use of transformations as a window into how students *think* about slope and intercepts, as well as how transformations can be used to foster an understanding of the difference between functions and equations, and variables and unknowns. They will use interactive tools to explore the link between graphic and symbolic representations of a function and study the object nature of functions. Learners will also examine how solving an equation is related to graphing a function and how a variable is related to an unknown. Finally they view videos of students performing the same activities (without the Linear Transformer) and discuss student thinking and teacher strategies, both as a teacher and as a learner.

Learners will:

Read

- *Meet the Students –Part 1* which provides background information about the students in the videos.
- *Assessing Proportional Reasoning* – Looking at examples of true proportional reasoning and techniques for assessing student thinking.
- *Recognizing Contrasting Strategies* -Examine other strategies that students may apply to proportional reasoning problems.
- *Meet the Students –Part 2* which provides background information about the students in the second set of videos Read about the students they will meet in the second set of videos.

View videos:

- *Observing Student Thinking, Part 1*: Observe students working on the Starburst problem
- *Specialist Commentary, Part 1*: Watch videos of mathematics specialist, Dr. James Kaput, describing the Starburst problem.

Write in Journal (not required):

- Review notes from experiences engaging in the Function Analyzer challenges and address the following:
  - What is the difference between evaluating a function and solving an equation? Which values are you finding in each case?
  - What connections do you see between an equation and a function?
  - How is the distinction between "unknown" and "variable" revealed by this exercise?
- Compare observations from two challenges using the function analyzer and address the following:
  - Did you tend to focus on one representation more than another (graphic vs. symbolic)? What did your observations tell you?
  - How did your methods differ, or were they the same? If you used a different method with each interactive, how did your thinking change as you found the solution a second time?

Participate in an online discussion:

- Share comments in the Discussion Board, Session 3, *Observing Student Thinking, Part 1.*
- Share your comments in the Discussion Board, Session 3, Specialist Commentary, Part 1.
- Share your comments in the Discussion Board, Session 3, *Functions and Solutions.*

Complete activities and assignments

- *What's In an Equation?* - Work through an activity using the Linear Transformer
- *Warm-up: The Function Analyze r*- Get to know the Function Analyzer software tool.
- *Functions and More Functions* - Use the Function Analyzer and the Linear Transformer to highlight how different *tools illuminate different issues.*

Session 4: Your Classroom

The focus of this session changes to investigate the connection between the algebra content of this course and what you do in your own classroom. Learners will view more videos of students thinking about problems, and commentary about using transformations, technology, and alternative formulas as a path to a deeper understanding of algebra. They will also make links between the content of this course and their curriculum.

Learners will:

Watch Video:

- *Observing Student Thinking, Part 2*: Watch videos of students doing more work with the Starburst problem.
- *Specialist Commentary, Part 2:* Watch videos of Dr. Kaput explaining some challenges of teaching algebra.

Write in Journal:

- Review curriculum materials and consider the following:
  - Where and how does your curriculum link symbolic and graphic representations of functions?
  - How does an understanding of the link between symbolic and graphic representations of functions relate to the standardized tests or other formal assessments your students must pass?
  - How, if at all, does your curriculum use the idea of transforming linear functions by translation, rotation, or reflection?
  - How can you bring transformations of linear functions into your classroom to help your students understand the links between symbolic and graphic forms? How would you adapt the ideas, approaches, and technology used in this course in your own classroom?
- Look at guiding questions to enhance the learning experience for students, Such as:
  - Within each challenge, what elements do the functions have in common? What elements are different?
  - As you move from one challenge to the next, what changes in the functions? What stays the same?

Participate in an online discussion:

- Share your comments in the Discussion Board, Session 4, *Observing Student Thinking, Part 2.*
- Share your comments in the Discussion Board, Session 4, *Links to the Curriculum.*

Complete activities and assignments:

- *Links to the Curriculum* : Compare your classroom and textbook approach to the one used here.
- *For Your Students*: Explore activities with your students.

Session 5: Your Plan

In this session, learners will look back over the landscape of ideas that were explored throughout this course. Also, in this last week, learners will create a final project to integrate the algebraic concepts developed throughout the course in their instructional program. They will then celebrate their achievements and say goodbye to classmates and facilitator.

Learners will:

Read:
- A review of major topics addressed in this course

Complete activities and assignments:
- Create either a lesson plan or action plan for applying what was learned to their instructional program:
  - Lesson plan – Select a specific activity (such as one of the "For Your Students" activities) that facilitates having students share mathematical ideas. Modify it to address the learning styles and characteristics of their students.
  - Action plan – Select a specific action or instructional strategy learner want to address, such as focusing on specific kinds of questions that elicit student thinking or specific personal activities to cultivate their listening skills

Participate in an online discussion:
- Post in the Discussion Board, Session 5, *Gallery of Plans-learners' completed plans*

Write in online journal (not required):
- Review written work and memories of what was learned and record a personal self-assessment and reflection.

Schedule

This course is designed is a 30 hour course to be held over 5 weeks. Learners will spend 4 to 6 hours per week to complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Requirements

Learners are expected to:
- Complete all assignments.
- Maintain an online journal.
- Participate and actively engage in discussions with fellow learners while contributing to the social construction of knowledge.
- Be self-directed and self-motivated.
- Ask for assistance when they need it.

Materials (hardware, software, plug-ins for Windows and Macintosh)

Operating System
For the best experience, use the newer operating systems: Mac OS X, Windows 98, Windows 2000 and Windows XP. Additional operating systems (for example Linux) appear to work, but are not tested. Mac OS 9 does not support a current version of Java, which is needed to use the interactives.

Browser
Use Internet Explorer, Mozilla, or Netscape with Windows operating systems. MAC users should use Netscape or Mozilla. Browser must have cookies enabled to support course login.

Video Players

One of the following video players is required in order to view the videos. Seeing Math recommends QuickTime.

- QuickTime
- RealPlayer
- Windows Media Player

Java

This course contains "interactives" — software applications that focus on one particular math concept. These require Java 1.3.1 or higher.

Word processor
Internet service provider
E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course can be taken for graduate credit on a pass/fail basis, or for a letter grade and graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

Last Update: December 2, 2005

Rubrics for Discussion

The assessment rubrics fall into two categories: discussions and activities. Learners read these rubrics to get the "big picture" perspective of what's expected. They then refer to them from time to time during the course to remind them of the target, and to use as a self-assessment tool.

In an online course, participation means posting. Most activities in this course require you to share your thoughts on a subject (such as a reading or a video), or to complete a hands-on assignment and discuss the experience with your peers. This collaboration leads to insights unavailable to individuals alone—we all learn together.

The facilitator will look for ***frequent*** and ***appropriate*** contributions to class discussions from all participants. "Frequent" means posting on at least three days each week. "Appropriate" is based on the level of contribution as a whole (rather than allotting specific points for content, style, particular solutions, etc.). The following characteristics make up an excellent body of discussion contributions:

- Is grounded in the ideas, readings, and activities of the course.
- Connects to and builds on the ideas of others, and advances the collective thinking about content and pedagogy.
- Shows respect for and integrates multiple views (even views that at first appear contradictory or unrelated).
- Achieves or reaches toward new insights about mathematics and teaching.
- Takes risks by sharing tentative or newly formed ideas, mistakes, or misconceptions.
- Expresses content clearly.
- Makes skillful connections between natural language, mathematical language, and student thinking.
- Elicits reflection and responses from other participants.
- Questions other participants in order to clarify and extend own ideas.

Rubric for Mathematical and Pedagogical Activities

Assignments ask learners to post written work in the course—for instance, when they solve a problem and describe their thought processes in working towards a solution. They are asked to wrestle with a math problem, interactive, or ideas. Then share this work with their facilitator and peers as a post in the Discussion Board.
The facilitator measures learners' effort, care, and understanding in reading and carrying out the assignments using the following criteria: The learner:

- Posts clear and detailed reports on assignments and observations of own learning processes.
- Focuses not on the "right answer," but on experiencing and observing learning processes.
- Makes connections among more than two representations (real-life, symbolic, graphic, numeric).
- Considers what different representations contribute to one's own and students' learning of algebra.
- Generates different real-life situations for the same mathematical setting, and conversely, generates different mathematical models to describe variations on the same real-life situation.
- Makes connections among mathematical concepts and describes them clearly.
- Explores the consequences of those connections to understanding and teaching mathematics.
- Clearly identifies, describes, and justifies the strategies used to solve problems.

While these rubrics may seem ambitious, learners are not required to meet every criteria for each assignment. The facilitator will apply individual criteria as necessary (for

instance, not all activities require learners to use multiple representations of math concepts). Learners use these as a general guide to gauge the quality of their work.

Learners are also encouraged to keep a journal of their thoughts and rough drafts which serves as an automatic record of their work.