Title

Seeing Math: Linear Functions

Target Audience

This course is intended for pre-service and in-service teachers of mathematics grades 6-12.

Prerequisites

To successfully participate and complete the assignments in this course, the learner must:

- Have knowledge of basic algebra
- Have comfort with graphing a line given its equation, finding the equation of a line given its graph, and creating a T-chart with data points from either an equation or a graph;
- Be familiar with the terms *domain*, *range*, and *interval.*

Course Description

Linear Functions introduces algebra through the mathematically cohesive concept of functions and grounds it by modeling real-life situations. Through problem solving, observation of students, discussion, and readings, learners:

- Explore a multifaceted, real world problem that generates varied approaches and solutions
- Distinguish linear functions from linear equations
- Interpret the slope of a linear function as a rate of change
- Identify ways in which multiple representations of functions can strengthen mathematical understanding
- Recognize the value of using piecewise functions to understand important characteristics of linear functions, such as domain and range

In addition, learners explore ideas about framing and teaching algebra, examine ways to support student understanding, and adapt a problem to enrich their own curriculum. Finally, they come away with a tangible benefit—interactive software and activities to use with students (along with alternative activities for classrooms that lack computer resources).

Instructor/Facilitator

See instructor/facilitator sheet

Credits

To be determined by college or university

Goals and Objectives

Interpret the meaning and characteristics of linear functions in the context of real-world situations. Learners will be able to:

- Distinguish between linear functions and linear equations through in-depth real-world modeling
- Identify and interpret the slope of a linear function as a rate of change
- Interpret domain and range in the context of a real-world situation

Identify ways in which multiple representations can express and enrich mathematical concepts. Learners will be able to:

- Use multiple representations of functions (symbolic, graphic, numerical, and verbal) in the same mathematical task
- Link representations in one format to representations in another

Use piecewise functions to accentuate characteristics of linearity in the context of real-world modeling situations. Learners will be able to:

- Represent a single piecewise function using multiple formulas
- Distinguish between different slopes within a single piecewise linear function
- Identify and interpret discontinuities within a single piecewise linear function, within the context of a modeling situation

Outline of Content and Assignments

After previewing the documents in the Course Information area, learners will proceed to Course Content to complete the five sessions, working through each session in order. Throughout the sessions, learners are asked to articulate their ideas in various forms: they are encouraged to reflect on their ideas and experiences in their online journal, and they take part in online discussions designed to allow them to glean information from other learners' experiences.

This five-week course is taken entirely over the Internet. Learners should expect to spend 4-6 hours per week completing assignments and discussions, and to log in to class and submit work or join discussions at least three times a week. Each week learners will complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Learners will also come away with a tangible benefit—interactive software and activities to use with students. These tools will be used within the course, such that learners are thoroughly familiar with them. In addition, learners are provided with alternative activities that do not require computers in case computer resources are not available for classroom use.

Session 1: Orientation

Much of the *Orientation* session is spent getting to know the course and meeting colleagues online. Learners also read about the approach to learning and teaching algebra implemented in this course.

Learners will:

Read

- Information about the syllabus, facilitator, rubrics and how work will be evaluated, and tips for successful online discussions.
- A summary of the goals and objectives for the course.
- *The Landscape of Learning* - A discussion on the underlying principles behind the learning in this course and what they should expect.
- *The Landscape of Algebra* – A discussion about the structure of functions that underlies algebra, and how this structure can help students make connections
- *Nouns, Verbs and Mathematics* - A discussion about the importance of being able to view mathematical expressions in more than one way.
- *Let's Discuss Approaches to Algebra* – Discussion questions related to the three primary reading assignments.

- *Journal* – About the value of keeping a record of one's own thoughts and problem solving during the course.

Complete activities and assignments
- Create personal homepage providing a biographical sketch.

Write in journal *(Note: Keeping a journal record is an ongoing assignment referred to within course activities, but the journal is not handed in or evaluated by the facilitator)*:
- Reflect on insights and ideas related to the three readings: *The Landscape of Learning, The Landscape of Algebra* and *Nouns, Verbs and Mathematics*.

Participate in online discussion
- Post messages in the Discussion Board, Session 1, *Getting to Know You.*
- After reading the questions in *Let's Discuss Approaches to Algebra*, share thoughts on *The Landscape of Learning*, *The Landscape of Algebra,* and *Nouns, Verbs* and *Mathematics* in the Discussion Board, Week 1, *Nouns, Verbs, and Landscapes.*

Session 2: Math Focus

This week learners will take a look at typical ways that algebra curricula approach linear functions, and explore these functions through a real world problem—by selecting cell phone plans for several clients. As they make decisions about various plans, they gain insight into their modes of problem solving. Learners will also try out, both as learner and teacher, different approaches and representations of the problem.

Finally, they will read a conversation comparing the practicality of arithmetic to the promise of algebra…and ponder how to teach algebra in more relevant ways.

Learners will:

Read:
- *Snapshots from the Curriculum* - Looking at textbook examples through the lens of linear functions.
- *Observing Your Processes* - Guidelines on observing personal problem solving.
- *Problem Solving Approaches* - An exploration of some approaches that the *Got a Plan*? problem generates (approaches learners may not have used the first time).
- *A Graphical Approach* - Observing the cell phone plans on a graph to reveal information visually.
- *Why Algebra?* - Looking at the challenges teachers face in making algebra relevant

Complete activities and assignments:
- *Diving In: Got a Plan?* - Solve problems that enable a learner representing a customer service representative to help customers choose plans that fit their needs.
- *Piecewise Grapher Warm-ups* – Activities to acquaint learners with the new software tool before using it in an activity

Write in journal-
- Reflect on approaches used to solve the *Diving In* problem in whatever way seemed natural to the learner and after reviewed from three viewpoints—numerical/tabular, symbolic, and graphic.
- Reflect on insights gained from reading *Why Algebra?* For example:
  - How do they relate to Clarissa's view of real world problems? In their experience, what are the benefits of using problems that allow for multiple solutions or multiple strategies? What are the challenges?

- In the *Diving In* and subsequent activities, they make comparisons graphically, make decisions, justify their reasoning, and interpret their solutions in the context of real world situations. Where do these skills fit into their notions of algebra?
- Jacob feels he'll need to teach about piecewise functions before using the *Got a Plan?* problem with his students. Some teachers like to teach a skill or concept and then apply it to real world situations. Other teachers like to use real life situations to motivate students, and to generate a need for the skill or concept. What is their experience?
- How do they relate to Donna's approach to the problem? What strategies do they use to make algebra meaningful for students?

Participate in an online discussion:

- Share solutions and comments in the Discussion Board, Session 2, *Approaches to "Got a Plan?"*
- Share comments in the Discussion Board, Week 2, *Why Algebra?*

Session 3: Student Thinking

This session takes a close look at student thinking. Learners watch videos of students as they work on the *Got a Plan?* cell phone problem. Learners discuss observations with colleagues, and hear the comments of specialist Dr. James Kaput on common student interpretations and the core mathematics in the videos.

Learners will:

Read:

- *Meet the Students* - Background information about the students in the videos.
- *Observing Student Thinking* – "Don't Miss" comments from the student video
- *Specialist Commentary: Student Thinking* - Summary of Dr. Kaput's commentary

View videos:

- *Observing Student Thinking* - Observe students working with on the *Got A Plan?* problem.
- *Specialist Commentary: Student Thinking* - Watch as mathematics specialist, Dr. Kaput, discusses the underlying mathematical themes in the students' work.

Write in journal

- Reflect on what they considered interesting or significant in the videos and their thoughts on the "Don't Miss" items. They choose two or three issues or questions that stood out to summarize.
- Reflect on what learners considered interesting or significant in the commentary by Dr. Kaput. In particular, learners focus the four themes presented: Mathematics is About Something, Rate of Change, Student Difficulties, and Graphs or Equations.

Participate in an online discussion:

- Post summaries of reflections from observing students on the Discussion Board, Session 3, *Observing Student Thinking.*
- Post summaries of reflections to Dr. Kaput's insights on the Discussion Board, Session 3, *Specialist Commentary, Student Thinking*.

Session 4: Your Classroom

In this session, learners will consider different real life situations that can be modeled by piecewise linear functions. They will hear ideas from specialist Dr. James Kaput, and explore situations other than the cell phone model that they can use to develop problems for their students. .

As they develop and adapt problems, they will use many of the ideas they have explored so far. They will also discuss the problem with colleagues and take advantage of each other's experience and ideas.

Learners will:

Read:

- *Specialist Commentary: Modeling* – Summary of problem underlying the commentator video

Complete activities and assignments:

- *Speed-Cell Promotions* – Using the Piecewise Linear Grapher, solve problem that extends the ideas and concepts in the *Got a Plan?* cell phone problem. Learners will continue to explore the ways that concepts arise from a real world application. In particular, will see how piecewise linear functions can highlight:
  - The language of domain and range
  - The ideas of continuity and discontinuity of functions
  - Standard characteristics of linear functions
- *Adapting Models for Your Curriculum* and *Adapting Models, continued* - Look at some real world situations that can be modeled by linear functions, and choose and adapt one problem for their classroom.
- *For Your Students* - Adapt the *Got a Plan?* activities that students worked on in the videos and try them in their classroom. This is an *optional* activity.

View video:

- *Specialist Commentary: Modeling* - Watch as mathematics specialist Dr. Kaput highlights the concepts that can be drawn from the cell phone and other models.

Write in journal-

- Reflect in journal on how the Piecewise Linear Grapher made the concepts in this activity more tangible. How might it enhance their students' understanding of the characteristics of linear functions? How might they use or adapt this activity for their classrooms?
- Consider, in more depth, the types of models they would like to use in developing their problems for their students, and the concepts that might arise through those models.

Participate in an online discussion:

- Share a summary of two or three ideas, observations or questions that stood out in the assignment, in the Discussion Board, Session 4, *Speed-Cell Promotions.* Add highlights from your responses to the “Think About It” questions.
- Post problems and the considerations they took into account as they adapted it, in the Discussion Board, Session 4, *Adapting Models for Your Curriculum*.

Session 5: Your Plan

In this session learners will look back over the landscape of ideas they explored during this course. They will review their thoughts and records to consolidate their learning experience, and create a project that integrates the algebraic concepts developed throughout the course into their teaching practice. Learners will also celebrate their achievements and say goodbye to their peers and facilitator.

Learners will:

Read:

- *Course Summary* - A review of major topics addressed in this course

Complete activities and assignments:

- *Your Classroom Plan* - Create either a lesson plan or action plan for applying what was learned to their instructional program:
  - Lesson plan – Select a specific activity (such as one of the "For Your Students" activities) that facilitates having students share mathematical ideas. Modify it to address the learning styles and characteristics of their students.
  - Action plan – Select a specific action or instructional strategy the learner wants to address, such as focusing on specific kinds of questions that elicit student thinking or specific personal activities to cultivate their listening skills
- *Self-Reflection* – Review their course notes on readings and assignments.
- *Aha! Oops! Whew!* – Continue reflecting on the course and choose significant moments of insight into math concepts or teaching ("aha," "oops," or "whew" experiences of learning) and consider how to take these into their teaching.

Write in journal

- Review written work and memories of what was learned and record a personal self-assessment and reflection.

Participate in online discussions:

- Post the Plan in the Discussion Board, Session 5, *Gallery of Plans*
- Share memorable course experiences and insights with colleagues in the Discussion Board, Session 5, *Aha, Oops, Whew!*

Schedule

This course is designed to be a 30 hour course conducted over 5 weeks. Learners will spend 4 to 6 hours per week to complete assignments such as solving problems, observing videos, reading, adapting problems for the classroom, and taking part in online discussions. In the last week of the course learners will focus on creating and completing a final project.

Requirements

Learners are expected to:

- Complete all assignments.
- Maintain an online journal.
- Participate and actively engage in discussions with fellow learners while contributing to the social construction of knowledge.
- Be self-directed and self-motivated.
- Ask for assistance when they need it.

Materials (hardware, software, plug-ins for Windows and Macintosh)

Operating System

For the best experience, use the newer operating systesm: Mac OS X, Windows 98, Windows 2000 and Windows XP. Additional operating systems (for example Linux) appear to work, but are not tested. Mac OS 9 does not support a current version of Java, which is needed to use the interactives.

Browser

Use Internet Explorer, Mozilla, or Netscape with Windows operating systems. MAC users should use Netscape or Mozilla. Browser must have cookies enabled to support course login.

Video Players

One of the following video players is required in order to view the videos. Seeing Math recommends QuickTime.

- QuickTime

- RealPlayer
- Windows Media Player

Java
This course contains "interactives" — software applications that focus on one particular math concept. These require Java 1.3.1 or higher.

Word processor
Internet service provider
E-mail

Academic Dishonesty Policy

To be inserted by university institution only

Evaluation

This course can be taken for graduate credit on a pass/fail basis, or for a letter grade and graduate credit. See graduate credit details pertaining to specific graduate credit institutions.

Last Update: December 2, 2005

Rubrics for Discussion

The assessment rubrics fall into two categories: discussions and activities. Learners read these rubrics to get the "big picture" perspective of what's expected. They then refer to them from time to time during the course to remind them of the target, and to use as a self-assessment tool.

In an online course, participation means posting. Most activities in this course require learners to share their thoughts on a subject (such as a reading or a video), or to complete a hands-on assignment and discuss the experience with peers. This collaboration leads to insights unavailable to individuals alone—we all learn together.

The facilitator will look for ***frequent*** and ***appropriate*** contributions to class discussions from all participants. "Frequent" means posting on at least three days each week. "Appropriate" is based on the level of contribution as a whole (rather than allotting specific points for content, style, particular solutions, etc.). The following characteristics make up an excellent body of discussion contributions:

- Is grounded in the ideas, readings, and activities of the course.
- Connects to and builds on the ideas of others, and advances the collective thinking about content and pedagogy.
- Shows respect for and integrates multiple views (even views that at first appear contradictory or unrelated).
- Achieves or reaches toward new insights about mathematics and teaching.
- Takes risks by sharing tentative or newly formed ideas, mistakes, or misconceptions.
- Expresses content clearly.
- Makes skillful connections between natural language, mathematical language, and student thinking.
- Elicits reflection and responses from other participants.
- Questions other participants in order to clarify and extend own ideas.

Rubric for Mathematical and Pedagogical Activities

Assignments ask learners to post written work in the course—for instance, when they solve a problem and describe their thought processes in working towards a solution. They are asked to wrestle with a math problem, interactive, or ideas. Then share this work with their facilitator and peers as a post in the Discussion Board.
The facilitator measures learners' effort, care, and understanding in reading and carrying out the assignments using the following criteria: The learner:

- Posts clear and detailed reports on assignments and observations of own learning processes.
- Focuses not on the "right answer," but on experiencing and observing learning processes.
- Makes connections among more than two representations (real-life, symbolic, graphic, numeric).
- Considers what different representations contribute to one's own and students' learning of algebra.
- Generates different real-life situations for the same mathematical setting, and conversely, generates different mathematical models to describe variations on the same real-life situation.
- Makes connections among mathematical concepts and describes them clearly.
- Explores the consequences of those connections to understanding and teaching mathematics.
- Clearly identifies, describes, and justifies the strategies used to solve problems.

While these rubrics may seem ambitious, learners are not required to meet every criteria for each assignment. The facilitator will apply individual criteria as necessary (for instance, not all activities

require learners to use multiple representations of math concepts). Learners use these as a general guide to gauge the quality of their work.

Learners are also encouraged to keep a journal of their thoughts and rough drafts which serves as an automatic record of their work.