

**Title**

The Concept of Function for Grades 9-12

**Target Audience**

This course is intended for pre-service and in-service high school teachers.

**Prerequisites**

- Knowledge of how to plot points on a graph.
- Ability to identify a pattern in a set of numbers, if one exists, and to express that pattern as an equation.
- Understanding of the concept of ratio as a relationship between two values with the same unit of measure.

**Course Description**

This course focuses on the concept of function, including theories of function as a process, as a mathematical object, and as a tool for describing and predicting. Learners will first solve mathematical problems and engage in activities that deepen their understanding of function. They then consider how students approach the concept of function and discuss instructional strategies. Learners will explore and reflect on the misconceptions students have about functions and the difficulties students face in studying functions. The content focuses on generalizing patterns, understanding relations, performing transformations of functions, and analyzing complex functions. As a final assignment, learners construct an effective approach to teaching the concept of function.

**Instructor/Facilitator**

See instructor/facilitator sheet

**Credits**

To be determined by college or university

**Objectives**

Learners will gain:

- A deeper understanding of the concept of function, including perspectives of function as a process, as a mathematical object, and as a tool for describing and predicting.
- A greater appreciation for and understanding of students' difficulties learning the concept of function, including their misconceptions of functions.
- Strategies for helping students internalize the concept of function with fewer misconceptions and for addressing misconceptions that do occur.

**Outline of Content and Assignments**

After previewing the course introductory information, learners will proceed to the Assignments section to complete the following six parts, working through each part in order. In these sessions, they will solve mathematical problems as a learner and then use that experience to reflect on the misconceptions these problems generate in students and discuss strategies for helping students overcome their misconceptions. Learners will also read NCTM's "Principles and Standards for School Mathematics" and solve sample problems related to functions that have appeared on state tests, so that they can correlate the mathematics they have explored in this course to the mathematics students are required to learn. As a final task, they will rewrite a portion of a math textbook coinciding with the teaching and learning of function and its importance to the mathematical education of high school students. The parts, or "sessions," are:

- Session 1: Orientation
- Session 2: The Definition of Function
- Session 3: Process versus Object
- Session 4: Functions as Tools for Predicting and Describing
- Session 5: Misconceptions about Functions
- Session 6: Final Project

**Part 1: Orientation**

Learners will:

- Watch an introductory video about preparing themselves to think like a learner as well as a teacher for this course.
- Reflect on expectations and prior knowledge for this content
- Explore how functions are presented on state assessment
- Introduce themselves to other participants.

Read

- "Functions and Relations: A Unifying Theme for School Algebra in Grades 9-12 and "Mathematical Discourse." These articles will familiarize learners with the most current research about the content and pedagogy related to the concept of function.
- "Mathematical Discourse" to understand the importance of the discussion board.

Watch

- Introductory Video

Explore

- State Assessments Interactive

Respond in an Online Journal

- Professional goals and expectations for this course
- Write about personal experiences about learning or teaching functions.

Participate in Online Discussions

- Introduce themselves in the Virtual Cafe
- Definitions what is a fruit and what is a vegetable.

**Part 2: The Definition of Function**



Learners will:

- Identify how their concept images of function differ from the concept definitions of function.
- Determine the important features of functions, including non-algebraic functions.
- Identify limitations imposed by the concept image on the full understanding of the function concept.

Read

- "Concept Images and Definitions." This reading discusses how concept images and the actual definition can shape student understanding in the mathematics classroom.

Explore

- The "Functions: Yes or No?" problem and decide which relationships are functions and consider what they believe makes a relationship a function.
- Domain and Range

Participate in an online discussion

- Read and post responses on the discussion board regarding how the learner's concept image impacted the way function was viewed during the previous activity.

Complete the Assignments

- A description of the domain and range; and how they relate to the concept of function.

### Part 3: Process versus Object

Learners will:

- Identify process or object perspectives in the definitions of function and in the uses of the concept.
- Discuss ways to address the gap between these perspectives as it pertains to the teaching and learning the concept of function.

Read

- About the difference between process and objects underscoring the concept of function.
- Definitions of "function."
- "Functions Definitions from Textbooks"

Explore

- The "Functions and Thought Processes" problem as a process for thinking more deeply about how the learner approaches the ideas behind the concept of function.
- The "Relationship D" problem as another example of working with functions.

Participate in an online discussion

- Read and post responses on the discussion board regarding how the learner could remedy the disconnect between the students' perspectives of functions and the teachers' perspectives.

Complete the Assignment



- Functions and Thought Processes, and Relationship D problem

Part 4: Functions as Tools for Predicting and Describing

Learners will:

- Use functions to describe trends and predict results.
- Discuss the potential of introducing the function concept to students through description and prediction tasks.

Read

- The section “Understand Patterns, Relations, and Functions” in *PSSM Algebra Standards*, to familiarize themselves with what students need to know and be able to do

Explore

- Solve the "Weather Balloon" problem.
- Explore the "Color Machine" interactive. Learners will drop one of the objects on the left into the machine until they know the following:
  - What the machine does.
  - What would happen if they put the highlighted color into the machine.

Watch

- “Student Reflections” and “Teacher Reflections” and take notes about how teacher Elyse Sakata introduced functions to her students and about some of the teaching strategies she found helpful. This video will help learners anticipate and identify student misconceptions around patterns.

Participate in an online discussion

- Read and post responses on the discussion board regarding how the learner views the benefits and limitations or pitfalls for introducing functions to students as models (tools for describing and predicting).

Complete the Assignment

- Weather Balloon and Color Machine

Part 5: Misconceptions about Functions

Learners will:

- Identify common misconceptions students have about functions.
- Identify strategies to address those misconceptions.

Read

- “Misconceptions Held by Students and Teachers” This reading gives some insights into commonly held student misconceptions or misunderstandings related to the concept of function.

Explore

- Explore the functions in "Commonly Mistaken Functions." For each, learners will think of one or more reasons why a student might answer incorrectly when asked if a function is represented.
- Consider the relationship using "polar coordinates."
- (Optional:) Transformations activity

Watch the videos

- "Is it a Function?," which shows students deciding whether  $y$  is a function of  $x$ . As learners watch, they should note what kinds of misconceptions the students have and how those misconceptions are addressed.
- "A Common Method," which shows several different groups of students as they decide whether numerical relationships such as the ones in Activity 2 are functions.

Participate in an online discussion

- By reading and posting responses on the discussion board, learners share some effective ways to address student misconceptions, focusing on ways to prevent them from occurring and ways to correct them after they have occurred.
- By reading and posting responses on the discussion board, the learner reflects on what was learned about functions in this course and how a current textbook helps or hinders students' understanding of functions.

#### Part 6: Final Project

The Learner will:

- Reflect on the mathematical concepts, processes, and skills that you have learned in the course.
- Analyze the method an existing textbook or curriculum uses to introduce functions.
- Describe improvements to that method.

Solve

- "Function Problems on State Assessments" Teaching functions in depth prepares students for state assessments, as well as gives them a deeper understanding of the mathematics.

Complete the Final Project

- For the final project for this course, learners will examine how their textbook introduces and develops the concept of function. They will write a 3-4 page paper that includes two parts:
  - A critique of how functions are presented. They will consider the groundwork being laid for the concept as well as how the term *function* is actually introduced.
  - A description of how to write and better organize an introduction to and development of the function concept for their students. This will include sample exercises or activities.

Criteria for the final project

- A description of how the textbook lays the groundwork for the function concept
- A description of how the textbook defines function, noting the extent to which the introduction uses a process or object perspective
- A short analysis of how effective they expect the textbook's development to be, with reasons for their expectations and conclusions

- Identification of activities or exercises that can be replaced or improved upon, with a justification for changing the existing material
- Descriptions of at least two replacement or supplementary activities that address the weaknesses in the existing material

Respond in an Online Journal

- Reflect on what was learned in the course. What one or two ideas do you take away from this course that you believe will be of the most use for you in your classroom, and why?
- Reflect on how to advance professional development

### **Schedule**

This course is scheduled to take approximately 15 hours to complete readings, activities, video, assignments, reflections and a final project.

### **Requirements**

Learners are expected to:

- Complete all assignments
- Maintain and online journal
- Participate regularly in discussion boards

### **Evaluation**

Pass/fail upon satisfactory completion of assignments and discussion board participation

### **Materials (hardware, software, plug-ins)**

Technical Requirements

- Word processor
- Internet service provider
- E-mail

### **Academic Dishonesty Policy**

To be inserted by university institution only