

PBS TeacherLine New York Developing Algebraic Thinking Grades 3 – 5

New York State Learning Standards: Mathematics, Science, and Technology - Standard 3

Students will:

- understand the concepts of and become proficient with the skills of mathematics;
- communicate and reason mathematically;
- become problem solvers by using appropriate tools and strategies;

through the integrated study of number sense and operations, algebra, geometry, measurement, and statistics and probability.

Number Sense and Operations Strand

Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.

Number Systems

- 3.N.1 Skip count by 25's, 50's, 100's to 1,000
- 3.N.2 Read and write whole numbers to 1,000
- 3.N.3 Compare and order numbers to 1,000
- 3.N.4 Understand the place value structure of the base ten number system: 10 ones = 1 ten, 10 tens = 1 hundred, 10 hundreds = 1 thousand
- 3.N.5 Use a variety of strategies to compose and decompose three-digit numbers
- 3.N.6 Use and explain the commutative property of addition and multiplication
- 3.N.7 Use 1 as the identity element for multiplication
- 3.N.8 Use the zero property of multiplication
- 3.N.9 Understand and use the associative property of addition
- 3.N.10 Develop an understanding of fractions as part of a whole unit and as parts of a collection
- 3.N.11 Use manipulatives, visual models, and illustrations to name and represent unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{10}$) as part of a whole or a set of objects
- 3.N.12 Understand and recognize the meaning of numerator and denominator in the symbolic form of a fraction
- 3.N.13 Recognize fractional numbers as equal parts of a whole
- 3.N.14 Explore equivalent fractions
- 3.N.15 Compare and order unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) and find their approximate locations on a number line
- 4.N.6 Understand, use, and explain the associative property of multiplication

Number Theory

- 3.N.16 Identify odd and even numbers
- 3.N.17 Develop an understanding of the properties of odd/even numbers as a result of addition or subtraction
- 4.N.13 Develop an understanding of the properties of odd/even numbers as a result of multiplication

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Students will understand meanings of operations and procedures, and how they relate to one another.

Operations

- 3.N.18 Use a variety of strategies to add and subtract 3-digit numbers (with and without regrouping)
- 3.N.19 Develop fluency with single-digit multiplication facts
- 3.N.20 Use a variety of strategies to solve multiplication problems with factors up to 12 x 12
- 3.N.21 Use the area model, tables, patterns, arrays, and doubling to provide meaning for multiplication
- 3.N.22 Demonstrate fluency and apply single-digit division facts
- 3.N.23 Use tables, patterns, halving, and manipulatives to provide meaning for division
- 3.N.24 Develop strategies for selecting the appropriate computational and operational method in problem solving situations

- 4.N.15 Select appropriate computational and operational methods to solve problems
- 4.N.16 Understand various meanings of multiplication and division
- 4.N.17 Use multiplication and division as inverse operations to solve problems

- 5.N.18 Evaluate an arithmetic expression using order of operations including multiplication, division, addition, subtraction and parentheses

Students will compute accurately and make reasonable estimates.

Estimation

- 3.N.25 Estimate numbers up to 500
- 3.N.26 Recognize real world situations in which an estimate (rounding) is more appropriate
- 3.N.27 Check reasonableness of an answer by using estimation

- 4.N.27 Check reasonableness of an answer by using estimation

- 5.N.27 Justify the reasonableness of answers using estimation

Algebra Strand

Students will represent and analyze algebraically a wide variety of problem solving situations.

Variables and Expressions

- 4.A.1 Evaluate and express relationships using open sentences with one operation

- 5.A.1 Define and use appropriate terminology when referring to constants, variables, and algebraic expressions
- 5.A.2 Translate simple verbal expressions into algebraic expressions

Students will perform algebraic procedures accurately.

Variables and Expressions

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5.A.3 Substitute assigned values into variable expressions and evaluate using order of operations

Equations and Inequalities

3.A.1 Use the symbols $<$, $>$, $=$ (with and without the use of a number line) to compare whole numbers and unit fractions ($\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{10}$)

4.A.2 Use the symbols $<$, $>$, $=$, and \square , (with and without the use of a number line) to compare whole numbers and unit fractions and decimals (up to hundredths)

4.A.3 Find the value or values that will make an open sentence true, if it contains $<$ or $>$

5.A.4 Solve simple one-step equations using basic whole-number facts

5.A.5 Solve and explain simple one-step equations using inverse operations involving whole numbers

5.A.6 Evaluate the perimeter formula for given input values

Students will recognize, use, and represent algebraically patterns, relations, and functions.

Patterns, Functions, and Relations

3.A.2 Describe and extend numeric (+, -) and geometric patterns

4.A.4 Describe, extend, and make generalizations about numeric (\div , \times , $-$, $+$, $,$, $,$, $,$) and geometric patterns

4.A.5 Analyze a pattern or a whole-number function and state the rule, given a table or an input/output box

Problem Solving Strand

Students will build new mathematical knowledge through problem solving.

3.PS.1 Explore, examine, and make observations about a social problem or mathematical situation

3.PS.2 Understand that some ways of representing a problem are more helpful than others

3.PS.3 Interpret information correctly, identify the problem, and generate possible solutions

4.PS.1 Explore, examine, and make observations about a social problem or mathematical situation

4.PS.2 Understand that some ways of representing a problem are more helpful than others

4.PS.3 Interpret information correctly, identify the problem, and generate possible solutions

5.PS.1 Know the difference between relevant and irrelevant information when solving problems

5.PS.2 Understand that some ways of representing a problem are more efficient than others

5.PS.3 Interpret information correctly, identify the problem, and generate possible strategies and solutions

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Students will solve problems that arise in mathematics and in other contexts.

- 3.PS.4 Act out or model with manipulatives activities involving mathematical content from literature
- 3.PS.5 Formulate problems and solutions from everyday situations
- 3.PS.6 Translate from a picture/diagram to a numeric expression
- 3.PS.7 Represent problem situations in oral, written, concrete, pictorial, and graphical forms
- 3.PS.8 Select an appropriate representation of a problem

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- 5.PS.4 Act out or model with manipulatives activities involving mathematical content from literature
- 5.PS.5 Formulate problems and solutions from everyday situations
- 5.PS.6 Translate from a picture/diagram to a numeric expression
- 5.PS.7 Represent problem situations verbally, numerically, algebraically, and/or graphically
- 5.PS.8 Select an appropriate representation of a problem
- 5.PS.9 Understand the basic language of logic in mathematical situations (and, or, not)

Students will apply and adapt a variety of appropriate strategies to solve problems.

- 3.PS.9 Use trial and error to solve problems
- 3.PS.10 Use process of elimination to solve problems
- 3.PS.11 Make pictures/diagrams of problems
- 3.PS.12 Use physical objects to model problems
- 3.PS.13 Work in collaboration with others to solve problems
- 3.PS.14 Make organized lists to solve numerical problems
- 3.PS.15 Make charts to solve numerical problems
- 3.PS.16 Analyze problems by identifying relationships
- 3.PS.17 Analyze problems by identifying relevant versus irrelevant information
- 3.PS.18 Analyze problems by observing patterns
- 3.PS.19 State a problem in their own words

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- 4.PS.17 Analyze problems by identifying relevant versus irrelevant information
- 4.PS.18 Analyze problems by observing patterns
- 4.PS.19 State a problem in their own words

- 5.PS.10 Work in collaboration with others to solve problems

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- 5.PS.11 Translate from a picture/diagram to a number or symbolic expression
- 5.PS.12 Use trial and error and the process of elimination to solve problems
- 5.PS.13 Model problems with pictures/diagrams or physical objects
- 5.PS.14 Analyze problems by observing patterns
- 5.PS.15 Make organized lists or charts to solve numerical problems

Students will monitor and reflect on the process of mathematical problem solving.

- 3.PS.20 Determine what information is needed to solve a problem
- 3.PS.21 Discuss with peers to understand a problem situation
- 3.PS.22 Discuss the efficiency of different representations of a problem
- 3.PS.23 Verify results of a problem
- 3.PS.24 Recognize invalid approaches
- 3.PS.25 Determine whether a solution is reasonable in the context of the original problem

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- 5.PS.16 Discuss with peers to understand a problem situation
- 5.PS.17 Determine what information is needed to solve problem
- 5.PS.18 Determine the efficiency of different representations of a problem
- 5.PS.19 Differentiate between valid and invalid approaches
- 5.PS.20 Understand valid counterexamples
- 5.PS.21 Explain the methods and reasoning behind the problem solving strategies used
- 5.PS.22 Discuss whether a solution is reasonable in the context of the original problem
- 5.PS.23 Verify results of a problem

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

- 3.RP.1 Use representations to support mathematical ideas
- 3.RP.2 Determine whether a mathematical statement is true or false and explain why

- 4.RP.1 Use representations to support mathematical ideas
- 4.RP.2 Determine whether a mathematical statement is true or false and explain why

- 5.RP.1 Recognize that mathematical ideas can be supported using a variety of strategies
- 5.RP.2 Understand that mathematical statements can be justified, using models, facts and relationships to explain their thinking

Students will make and investigate mathematical conjectures.

- 3.RP.3 Investigate the use of knowledgeable guessing by generalizing mathematical ideas
- 3.RP.4 Make conjectures from a variety of representations

- 4.RP.3 Investigate the use of knowledgeable guessing by generalizing mathematical ideas

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4.RP.4 Make conjectures from a variety of representations

5.RP.3 Investigate conjectures, using arguments and appropriate mathematical terms

5.RP.4 Make and evaluate conjectures, using a variety of strategies

Students will develop and evaluate mathematical arguments and proofs.

3.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions

3.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms

3.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

4.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions

4.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms

4.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

5.RP.5 Justify general claims or conjectures, using manipulatives, models, expressions, and mathematical relationships

5.RP.6 Develop and explain an argument verbally, numerically, and/or graphically

5.RP.7 Verify claims other students make, using examples and counterexamples when appropriate

Students will select and use various types of reasoning and methods of proof.

3.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions

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4.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

5.RP.5 Justify general claims or conjectures, using manipulatives, models, expressions, and mathematical relationships

5.RP.6 Develop and explain an argument verbally, numerically, and/or graphically

5.RP.7 Verify claims other students make, using examples and counterexamples when appropriate

Students will select and use various types of reasoning and methods of proof.

4.RP.8 Justify an argument by trying many cases

4.RP.9 Disprove an argument by finding counterexamples

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5.RP.8 Justify an argument through examples/counterexamples and special cases

Communication Strand

Students will organize and consolidate their mathematical thinking through communication.

3.CM.1 Understand and explain how to organize their thought process

3.CM.2 Verbally explain their rationale for strategy selection

3.CM.3 Provide reasoning both in written and verbal form

4.CM.1 Understand and explain how to organize their thought process

4.CM.2 Verbally explain their rationale for strategy selection

4.CM.3 Provide reasoning both in written and verbal form

5.CM.1 Provide an organized thought process that is correct, complete, coherent, and clear

5.CM.2 Explain a rationale for strategy selection

5.CM.3 Organize and accurately label work

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

3.CM.4 Organize and accurately label work

3.CM.5 Share organized mathematical ideas through the manipulation of objects, drawings, pictures, charts, graphs, tables, diagrams, models, symbols, and expressions in written and verbal form

3.CM.6 Answer clarifying questions from others

4.CM.4 Organize and accurately label work

4.CM.5 Share organized mathematical ideas through the manipulation of objects, drawing, pictures, charts, graphs, tables, diagrams, models, symbols, and expressions in written and verbal form

4.CM.6 Answer clarifying questions from others

5.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models, and symbols in written and verbal form

5.CM.5 Answer clarifying questions from others

Students will analyze and evaluate the mathematical thinking and strategies of others.

3.CM.7 Listen for understanding of mathematical solutions shared by other students

3.CM.8 Consider strategies used and solutions found in relation to their own work

4.CM.7 Restate mathematical solutions shared by other students

4.CM.8 Consider strategies used and solutions found in relation to their own work

5.CM.6 Understand mathematical solutions shared by other students

5.CM.7 Raise questions that elicit, extend, or challenge others' thinking

5.CM.8 Consider strategies used and solutions found by others in relation to their own work

Students will use the language of mathematics to express mathematical ideas precisely.

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- 3.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 3.CM.10 Describe objects, relationships, solutions and rationale using appropriate vocabulary
- 3.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

- 4.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 4.CM.10 Describe objects, relationships, solutions, and rationale using appropriate vocabulary
- 4.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

- 5.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
- 5.CM.10 Use appropriate vocabulary when describing objects, relationships, mathematical solutions, and rationale
- 5.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

Connections Strand

Students will recognize and use connections among mathematical ideas.

- 3.CN.1 Recognize, understand, and make connections in their everyday experiences to mathematical ideas
- 3.CN.2 Compare and contrast mathematical ideas
- 3.CN.3 Connect and apply mathematical information to solve problems

- 4.CN.1 Recognize, understand, and make connections in their everyday experiences to mathematical ideas
- 4.CN.2 Compare and contrast mathematical ideas
- 4.CN.3 Connect and apply mathematical information to solve problems

- 5.CN.1 Understand and make connections and conjectures in their everyday experiences to mathematical ideas
- 5.CN.2 Explore and explain the relationship between mathematical ideas
- 5.CN.3 Connect and apply mathematical information to solve problems

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

- 3.CN.4 Understand multiple representations and how they are related
- 3.CN.5 Model situations with objects and representations and be able to make observations

- 4.CN.4 Understand multiple representations and how they are related
- 4.CN.5 Model situations with objects and representations and be able to make observations

- 5.CN.4 Understand multiple representations and how they are related

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5.CN.5 Model situations with objects and representations and be able to draw conclusions

Students will recognize and apply mathematics in contexts outside of mathematics.

3.CN.6 Recognize the presence of mathematics in their daily lives
3.CN.7 Apply mathematics to solve problems that develop outside of mathematics
3.CN.8 Recognize and apply mathematics to other disciplines

4.CN.6 Recognize the presence of mathematics in their daily lives
4.CN.7 Apply mathematics to solve problems that develop outside of mathematics
4.CN.8 Recognize and apply mathematics to other disciplines

5.CN.6 Recognize and provide examples of the presence of mathematics in their daily lives
5.CN.7 Apply mathematics to problem situations that develop outside of mathematics
5.CN.8 Investigate the presence of mathematics in careers and areas of interest
5.CN.9 Recognize and apply mathematics to other disciplines and areas of interest

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

3.R.1 Use verbal and written language, physical models, drawing charts, graphs, tables, symbols, and equations as representations
3.R.2 Share mental images of mathematical ideas and understandings
3.R.3 Recognize and use external mathematical representations
3.R.4 Use standard and nonstandard representations with accuracy and detail

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4.R.2 Share mental images of mathematical ideas and understandings
4.R.3 Recognize and use external mathematical representations
4.R.4 Use standard and nonstandard representations with accuracy and detail

5.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, and technology as representations
5.R.2 Explain, describe, and defend mathematical ideas using representations
5.R.3 Read, interpret, and extend external models
5.R.4 Use standard and nonstandard representations with accuracy and detail

Students will select, apply, and translate among mathematical representations to solve problems.

3.R.5 Understand similarities and differences in representations.
3.R.6 Connect mathematical representations with problem solving
3.R.7 Construct effective representations to solve problems

4.R.5 Understand similarities and differences in representations
4.R.6 Connect mathematical representations with problem solving
4.R.7 Construct effective representations to solve problems

5.R.5 Use models to explore problem situations

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5.R.6 Investigate relationships between different representations and their impact on a given problem

Students will use representations to model and interpret physical, social, and mathematical phenomena.

3.R.8 Use mathematics to show and understand physical phenomena (i.e., estimate and represent the number of apples in a tree)

3.R.9 Use mathematics to show and understand social phenomena (i.e., determine the number of buses required for a field trip)

3.R.10 Use mathematics to show and understand mathematical phenomena (i.e., use a multiplication grid to solve odd and even number problems)

4.R.8 Use mathematics to show and understand physical phenomena (i.e., estimate and represent the number of apples in a tree)

4.R.9 Use mathematics to show and understand social phenomena (i.e., determine the number of buses required for a field trip)

4.R.10 Use mathematics to show and understand mathematical phenomena (i.e., use a multiplication grid to solve odd and even number problems)

5.R.7 Use mathematics to show and understand physical phenomena (i.e., determine the perimeter of a bulletin board)

5.R.8 Use mathematics to show and understand social phenomena (i.e., construct tables to organize data showing book sales)

5.R.9 Use mathematics to show and understand mathematical phenomena (i.e., find the missing value that makes the equation true: $(3 + 4) + 5 = 3 + (4 + \underline{\quad})$)