PBS TeacherLine New York
Fostering Cooperative Learning, Discussion, & Critical Thinking in Elementary Math
Grades 1 – 5

New York State Learning Standards: Mathematics, Science, and Technology - Standard 3

Students will:

- understand the concepts of and become proficient with the skills of mathematics;
- communicate and reason mathematically;
- become problem solvers by using appropriate tools and strategies;

through the integrated study of number sense and operations, algebra, geometry, measurement, and statistics and probability.

Problem Solving Strand

Students will build new mathematical knowledge through problem solving.

1.PS.1 Explore, examine, and make observations about a social problem or mathematical situation
1.PS.2 Interpret information correctly, identify the problem, and generate possible solutions

2.PS.1 Explore, examine, and make observations about a social problem or mathematical situation
2.PS.2 Interpret information correctly, identify the problem, and generate possible solutions

3.PS.1 Explore, examine, and make observations about a social problem or mathematical situation
3.PS.2 Understand that some ways of representing a problem are more helpful than others
3.PS.3 Interpret information correctly, identify the problem, and generate possible solutions

4.PS.1 Explore, examine, and make observations about a social problem or mathematical situation
4.PS.2 Understand that some ways of representing a problem are more helpful than others
4.PS.3 Interpret information correctly, identify the problem, and generate possible solutions

5.PS.1 Know the difference between relevant and irrelevant information when solving problems
5.PS.2 Understand that some ways of representing a problem are more efficient than others
5.PS.3 Interpret information correctly, identify the problem, and generate possible strategies and solutions

Students will solve problems that arise in mathematics and in other contexts.

1.PS.3 Act out or model with manipulatives activities involving mathematical content from literature and/or story telling

1.PS.4 Formulate problems and solutions from everyday situations (i.e., counting the number of children in the class or using the calendar to teach counting)

2.PS.3 Act out or model with manipulatives activities involving mathematical content from literature and/or story telling
2.PS.4 Formulate problems and solutions from everyday situations (i.e., counting the number of children in the class, using the calendar to teach counting).

3.PS.4 Act out or model with manipulatives activities involving mathematical content from literature
3.PS.5 Formulate problems and solutions from everyday situations
3.PS.6 Translate from a picture/diagram to a numeric expression
3.PS.7 Represent problem situations in oral, written, concrete, pictorial, and graphical forms
3.PS.8 Select an appropriate representation of a problem

4.PS.4 Act out or model with manipulatives activities involving mathematical content from literature
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5.PS.5 Formulate problems and solutions from everyday situations
5.PS.6 Translate from a picture/diagram to a numeric expression
5.PS.7 Represent problem situations verbally, numerically, algebraically, and/or graphically
5.PS.8 Select an appropriate representation of a problem
5.PS.9 Understand the basic language of logic in mathematical situations (and, or, not)

Students will apply and adapt a variety of appropriate strategies to solve problems.

1.PS.5 Use informal counting strategies to find solutions
1.PS.6 Experience teacher-directed questioning process to understand problems
1.PS.7 Compare and discuss ideas for solving a problem with teacher and/or students to justify their thinking
1.PS.8 Use manipulatives (i.e., tiles, blocks) to model the action in problems
1.PS.9 Use drawings/pictures to model the action in problems

2.PS.5 Use informal counting strategies to find solutions
2.PS.6 Experience teacher-directed questioning process to understand problems
2.PS.7 Compare and discuss ideas for solving a problem with teacher and/or students to justify their thinking
2.PS.8 Use manipulatives (i.e., tiles, blocks) to model the action in problems
2.PS.9 Use drawings/pictures to model the action in problems

3.PS.9 Use trial and error to solve problems
3.PS.10 Use process of elimination to solve problems
3.PS.11 Make pictures/diagrams of problems

3.PS.12 Use physical objects to model problems
3.PS.13 Work in collaboration with others to solve problems
3.PS.14 Make organized lists to solve numerical problems
3.PS.15 Make charts to solve numerical problems
3.PS.16 Analyze problems by identifying relationships
3.PS.17 Analyze problems by identifying relevant versus irrelevant information
3.PS.18 Analyze problems by observing patterns
3.PS.19 State a problem in their own words

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4.PS.12 Use physical objects to model problems
4.PS.13 Work in collaboration with others to solve problems
4.PS.14 Make organized lists to solve numerical problems
4.PS.15 Make charts to solve numerical problems
4.PS.16 Analyze problems by identifying relationships
4.PS.17 Analyze problems by identifying relevant versus irrelevant information
4.PS.18 Analyze problems by observing patterns
4.PS.19 State a problem in their own words

5.PS.10 Work in collaboration with others to solve problems
5.PS.11 Translate from a picture/diagram to a number or symbolic expression
5.PS.12 Use trial and error and the process of elimination to solve problems
5.PS.13 Model problems with pictures/diagrams or physical objects
5.PS.14 Analyze problems by observing patterns
5.PS.15 Make organized lists or charts to solve numerical problems

Students will monitor and reflect on the process of mathematical problem solving.

1.PS.10 Explain to others how a problem was solved, giving strategies and justifications

2.PS.10 Explain to others how a problem was solved, giving strategies and justifications

3.PS.20 Determine what information is needed to solve a problem
3.PS.21 Discuss with peers to understand a problem situation
3.PS.22 Discuss the efficiency of different representations of a problem
3.PS.23 Verify results of a problem
3.PS.24 Recognize invalid approaches
3.PS.25 Determine whether a solution is reasonable in the context of the original problem

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4.PS.22 Discuss the efficiency of different representations of a problem
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4.PS.25 Determine whether a solution is reasonable in the context of the original problem

5.PS.16 Discuss with peers to understand a problem situation
5.PS.17 Determine what information is needed to solve problem
5.PS.18 Determine the efficiency of different representations of a problem
5.PS.19 Differentiate between valid and invalid approaches

5.PS.20 Understand valid counterexamples
5.PS.21 Explain the methods and reasoning behind the problem solving strategies used
5.PS.22 Discuss whether a solution is reasonable in the context of the original problem
5.PS.23 Verify results of a problem

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

1.RP.1 Understand that mathematical statements can be true or false
1.RP.2 Recognize that mathematical ideas need to be supported by evidence

2.RP.1 Understand that mathematical statements can be true or false
2.RP.2 Recognize that mathematical ideas need to be supported by evidence

3.RP.1 Use representations to support mathematical ideas
3.RP.2 Determine whether a mathematical statement is true or false and explain why

4.RP.1 Use representations to support mathematical ideas
4.RP.2 Determine whether a mathematical statement is true or false and explain why

5.RP.1 Recognize that mathematical ideas can be supported using a variety of strategies
5.RP.2 Understand that mathematical statements can be justified, using models, facts and relationships to explain their thinking

Students will make and investigate mathematical conjectures.

1.RP.3 Investigate the use of knowledgeable guessing as a mathematical tool
1.RP.4 Explore guesses, using a variety of objects and manipulatives

2.RP.3 Investigate the use of knowledgeable guessing as a mathematical tool
2.RP.4 Explore guesses, using a variety of objects and manipulatives

3.RP.3 Investigate the use of knowledgeable guessing by generalizing mathematical ideas
3.RP.4 Make conjectures from a variety of representations

4.RP.3 Investigate the use of knowledgeable guessing by generalizing mathematical ideas
4.RP.4 Make conjectures from a variety of representations

5.RP.3 Investigate conjectures, using arguments and appropriate mathematical terms
5.RP.4 Make and evaluate conjectures, using a variety of strategies

Students will develop and evaluate mathematical arguments and proofs.

1.RP.5 Justify general claims, using manipulatives
1.RP.6 Develop and explain an argument verbally or with objects
1.RP.7 Listen to and discuss claims other students make

2.RP.5 Justify general claims, using manipulatives
2.RP.6 Develop and explain an argument verbally or with objects
2.RP.7 Listen to and discuss claims other students make

3.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions
3.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms
3.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

4.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions
4.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms
4.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

5.RP.5 Justify general claims or conjectures, using manipulatives, models, expressions, and mathematical relationships
5.RP.6 Develop and explain an argument verbally, numerically, and/or graphically
5.RP.7 Verify claims other students make, using examples and counterexamples when appropriate

Students will select and use various types of reasoning and methods of proof.

2.RP.8 Use trial and error strategies to verify claims

3.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions
3.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms
3.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

4.RP.5 Justify general claims or conjectures, using manipulatives, models, and expressions
4.RP.6 Develop and explain an argument using oral, written, concrete, pictorial, and/or graphical forms
4.RP.7 Discuss, listen, and make comments that support or reject claims made by other students

5.RP.5 Justify general claims or conjectures, using manipulatives, models, expressions, and mathematical relationships
5.RP.6 Develop and explain an argument verbally, numerically, and/or graphically
5.RP.7 Verify claims other students make, using examples and counterexamples when appropriate

Students will select and use various types of reasoning and methods of proof.

4.RP.8 Justify an argument by trying many cases
4.RP.9 Disprove an argument by finding counterexamples

5.RP.8 Justify an argument through examples/counterexamples and special cases

Communication Strand

Students will organize and consolidate their mathematical thinking through communication.

1.CM.1 Understand how to organize their thought processes with teacher guidance
1.CM.2 Verbally support their reasoning and answer

2.CM.1 Understand how to organize their thought processes
2.CM.2 Verbally support their reasoning and answer

3.CM.1 Understand and explain how to organize their thought process
3.CM.2 Verbally explain their rationale for strategy selection
3.CM.3 Provide reasoning both in written and verbal form

4.CM.1 Understand and explain how to organize their thought process
4.CM.2 Verbally explain their rationale for strategy selection
4.CM.3 Provide reasoning both in written and verbal form

5.CM.1 Provide an organized thought process that is correct, complete, coherent, and clear
5.CM.2 Explain a rationale for strategy selection
5.CM.3 Organize and accurately label work

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

1.CM.3 Share mathematical ideas through the manipulation of objects, drawings, pictures, charts, and symbols in both written and verbal explanations

2.CM.3 Share mathematical ideas through the manipulation of objects, drawings, pictures, charts, and symbols in both written and verbal explanations
3.CM.4 Organize and accurately label work
3.CM.5 Share organized mathematical ideas through the manipulation of objects, drawings, pictures, charts, graphs, tables, diagrams, models, symbols, and expressions in written and verbal form
3.CM.6 Answer clarifying questions from others

4.CM.4 Organize and accurately label work
4.CM.5 Share organized mathematical ideas through the manipulation of objects, drawing, pictures, charts, graphs, tables, diagrams, models, symbols, and expressions in written and verbal form
4.CM.6 Answer clarifying questions from others

5.CM.4 Share organized mathematical ideas through the manipulation of objects, numerical tables, drawings, pictures, charts, graphs, tables, diagrams, models, and symbols in written and verbal form
5.CM.5 Answer clarifying questions from others

Students will analyze and evaluate the mathematical thinking and strategies of others.

1.CM.4 Listen to solutions shared by other students
1.CM.5 Formulate mathematically relevant questions

2.CM.4 Listen to solutions shared by other students
2.CM.5 Formulate mathematically relevant questions

3.CM.7 Listen for understanding of mathematical solutions shared by other students
3.CM.8 Consider strategies used and solutions found in relation to their own work

4.CM.7 Restate mathematical solutions shared by other students
4.CM.8 Consider strategies used and solutions found in relation to their own work

5.CM.6 Understand mathematical solutions shared by other students
5.CM.7 Raise questions that elicit, extend, or challenge others' thinking
5.CM.8 Consider strategies used and solutions found by others in relation to their own work

Students will use the language of mathematics to express mathematical ideas precisely.

1.CM.6 Use appropriate mathematical terms, vocabulary, and language

2.CM.6 Use appropriate mathematical terms, vocabulary, and language

3.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
3.CM.10 Describe objects, relationships, solutions and rationale using appropriate vocabulary
3.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

4.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
4.CM.10 Describe objects, relationships, solutions, and rationale using appropriate vocabulary
4.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

5.CM.9 Increase their use of mathematical vocabulary and language when communicating with others
5.CM.10 Use appropriate vocabulary when describing objects, relationships, mathematical solutions, and rationale
5.CM.11 Decode and comprehend mathematical visuals and symbols to construct meaning

Connections Strand

Students will recognize and use connections among mathematical ideas.

1.CN.1 Recognize the connections of patterns in their everyday experiences to mathematical ideas
1.CN.2 Understand the connections between numbers and the quantities they represent
1.CN.3 Compare the similarities and differences of mathematical ideas

2.CN.1 Recognize the connections of patterns in their everyday experiences to mathematical ideas
2.CN.2 Understand and use the connections between numbers and the quantities they represent to solve problems
2.CN.3 Compare the similarities and differences of mathematical ideas

3.CN.1 Recognize, understand, and make connections in their everyday experiences to mathematical ideas
3.CN.2 Compare and contrast mathematical ideas
3.CN.3 Connect and apply mathematical information to solve problems

4.CN.1 Recognize, understand, and make connections in their everyday experiences to mathematical ideas
4.CN.2 Compare and contrast mathematical ideas
4.CN.3 Connect and apply mathematical information to solve problems

5.CN.1 Understand and make connections and conjectures in their everyday experiences to mathematical ideas
5.CN.2 Explore and explain the relationship between mathematical ideas
5.CN.3 Connect and apply mathematical information to solve problems

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

1.CN.4 Understand how models of situations involving objects, pictures, and symbols relate to mathematical ideas
1.CN.5 Understand meanings of operations and how they relate to one another
1.CN.6 Understand how mathematical models represent quantitative relationships

2.CN.4 Understand how models of situations involving objects, pictures, and symbols relate to mathematical ideas
2.CN.5 Understand meanings of operations and how they relate to one another
2.CN.6 Understand how mathematical models represent quantitative relationships

3.CN.4 Understand multiple representations and how they are related
3.CN.5 Model situations with objects and representations and be able to make observations

4.CN.4 Understand multiple representations and how they are related
4.CN.5 Model situations with objects and representations and be able to make observations

5.CN.4 Understand multiple representations and how they are related
5.CN.5 Model situations with objects and representations and be able to draw conclusions

Students will recognize and apply mathematics in contexts outside of mathematics.

1.CN.7 Recognize the presence of mathematics in their daily lives
1.CN.8 Recognize and apply mathematics to solve problems
1.CN.9 Recognize and apply mathematics to objects, pictures, and symbols

2.CN.7 Recognize the presence of mathematics in their daily lives
2.CN.8 Recognize and apply mathematics to solve problems
2.CN.9 Recognize and apply mathematics to objects, pictures and symbols

3.CN.6 Recognize the presence of mathematics in their daily lives
3.CN.7 Apply mathematics to solve problems that develop outside of mathematics
3.CN.8 Recognize and apply mathematics to other disciplines

4.CN.6 Recognize the presence of mathematics in their daily lives
4.CN.7 Apply mathematics to solve problems that develop outside of mathematics
4.CN.8 Recognize and apply mathematics to other disciplines

5.CN.6 Recognize and provide examples of the presence of mathematics in their daily lives
5.CN.7 Apply mathematics to problem situations that develop outside of mathematics
5.CN.8 Investigate the presence of mathematics in careers and areas of interest
5.CN.9 Recognize and apply mathematics to other disciplines and areas of interest

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

1.R.1 Use multiple representations including verbal and written language, acting out or modeling a situation, drawings, and/or symbols as representations
1.R.2 Share mental images of mathematical ideas and understandings
1.R.3 Use standard and nonstandard representations

2.R.1 Use multiple representations, including verbal and written language, acting out or modeling a situation, drawings, and/or symbols as representations
2.R.2 Share mental images of mathematical ideas and understandings
2.R.3 Use standard and nonstandard representations

3.R.1 Use verbal and written language, physical models, drawing charts, graphs, tables, symbols, and equations as representations
3.R.2 Share mental images of mathematical ideas and understandings
3.R.3 Recognize and use external mathematical representations
3.R.4 Use standard and nonstandard representations with accuracy and detail

4.R.1 Use verbal and written language, physical models, drawing charts, graphs, tables, symbols, and equations as representations
4.R.2 Share mental images of mathematical ideas and understandings
4.R.3 Recognize and use external mathematical representations
4.R.4 Use standard and nonstandard representations with accuracy and detail

5.R.1 Use physical objects, drawings, charts, tables, graphs, symbols, equations, and technology as representations
5.R.2 Explain, describe, and defend mathematical ideas using representations
5.R.3 Read, interpret, and extend external models
5.R.4 Use standard and nonstandard representations with accuracy and detail

Students will select, apply, and translate among mathematical representations to solve problems.

1.R.4 Connect mathematical representations with problem solving

2.R.4 Connect mathematical representations with problem solving

3.R.5 Understand similarities and differences in representations.
3.R.6 Connect mathematical representations with problem solving
3.R.7 Construct effective representations to solve problems

4.R.5 Understand similarities and differences in representations
4.R.6 Connect mathematical representations with problem solving
4.R.7 Construct effective representations to solve problems

5.R.5 Use models to explore problem situations
5.R.6 Investigate relationships between different representations and their impact on a given problem

Students will use representations to model and interpret physical, social, and mathematical phenomena.

1.R.5 Use mathematics to show and understand physical phenomena (i.e., estimate and represent the number of apples in a tree)
1.R.6 Use mathematics to show and understand social phenomena (i.e., count and represent sharing cookies between friends)
1.R.7 Use mathematics to show and understand mathematical phenomena (i.e., draw pictures to show a story problem, show number value using fingers on your hand)

2.R.5 Use mathematics to show and understand physical phenomena (i.e., estimate and represent the number of apples in a tree)
2.R.6 Use mathematics to show and understand social phenomena (i.e., count and represent sharing cookies between friends)
2.R.7 Use mathematics to show and understand mathematical phenomena (i.e., draw pictures to show a story problem or show number value using fingers on your hand)

3.R.8 Use mathematics to show and understand physical phenomena (i.e., estimate and represent the number of apples in a tree)
3.R.9 Use mathematics to show and understand social phenomena (i.e., determine the number of buses required for a field trip)
3.R.10 Use mathematics to show and understand mathematical phenomena (i.e., use a multiplication grid to solve odd and even number problems)

4.R.8 Use mathematics to show and understand physical phenomena (i.e., estimate and represent the number of apples in a tree)
4.R.9 Use mathematics to show and understand social phenomena (i.e., determine the number of buses required for a field trip)

4.R.10 Use mathematics to show and understand mathematical phenomena (i.e., use a multiplication grid to solve odd and even number problems)

5.R.7 Use mathematics to show and understand physical phenomena (i.e., determine the perimeter of a bulletin board)
5.R.8 Use mathematics to show and understand social phenomena (i.e., construct tables to organize data showing book sales)
5.R.9 Use mathematics to show and understand mathematical phenomena (i.e., find the missing value that makes the equation true: (3 + 4) + 5 = 3 + (4 + ___)